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ELEMENTARY

DESCRIPTIVE GEOMETRY

BY

C. H. McLEOD
McGill University, Montreal

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PREFACE.

The object of this volume is to put into concise form what the Author considers the minimum requirement of knowledge of the subject of Descriptive Geometry for students entering on an engineering course.

It is desirable where possible that the text should be supplemented by blackboard demonstration and individual classroom instruction.

The student should bear in mind that a thorough understanding of the methods of descriptive geometry is necessary to the proper interpretation of constructive drawing, and that in his study of the subject he should aim at grasping the methods rather than at learning special text-book problems.

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ELEMENTARY DESCRIPTIVE GEOMETRY.

INTRODUCTION.

UNDER *Descriptive Geometry* are included all methods of graphically representing lines, planes, and solids upon plane surfaces.

The special portion of the subject treated in the following chapters is rectangular or orthographic projection, with such incidental references to the more general methods of descriptive geometry as may be necessary to render the special problems, which are considered, intelligible.

In *Orthographic Projection* there are two projection planes, which are supposed to be at right angles to each other. The projection of a point is the intersection of the perpendicular from the point upon the plane. A point is, therefore, completely represented with reference to the projection planes by its two projections. The projection planes may be in any position, but they are usually supposed to be horizontal and vertical, and projections upon the two are distinguished accordingly. The objects to be projected may be in any position, and may, therefore, occupy any one of the quadrants or dihedral angles formed by the planes. The vertical projection is also sometimes called the *elevation,* and the horizontal projection the *plan.* The line in which the planes intersect will be distinguished as XY. When the planes are turned about this line until they come together, the portion above the horizontal line is conceived to be the vertical plane above the horizontal, as XLMY in the sketch, Fig. 1, and the horizontal plane behind the vertical, as XSRY. Similarly the portion below XY represents the horizontal in front of the vertical, as XPQY, and the vertical below the horizontal, as XNOY.

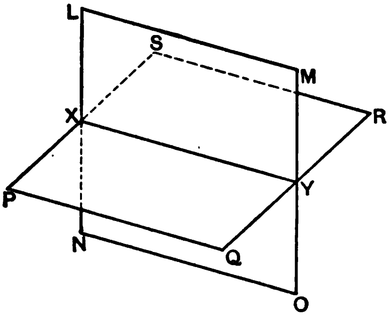


Fig. 1.

The surface of the drawing-paper, therefore, represents everywhere two planes, and the planes extend indefinitely in all directions.

Notation.—Points are represented by capital letters, horizontal projections of points by small italics, and vertical projections by the same accented. Thus A is a point, *a* its horizontal, and *a'* its vertical projection. Similarly, if the extremities of a line are A and B, AB represents the line, *ab* its plan, and *a'b'* its elevation. Where there are several projection planes for the same figure and a given point A is projected on them, the several plans may be denoted by a_1, a_2, etc., and the several elevations by a'_1, a'_2, etc.

The vertical and horizontal planes may be denoted respectively by V.P. and H.P., and XY will, in all cases, stand for the intersection line of these planes.

CHAPTER I.

POINTS AND LINES.

STRAIGHT lines are represented either by their projections or by their *traces*. The traces of a line are the points where the line or its production meets the projection planes.

Lines which are parallel have their projections on the same planes parallel. When a line is parallel to a projection plane, a projection of the line is parallel to XY.

PROBLEM I.—Given the position of a point with reference to the projection planes, to determine its projections.

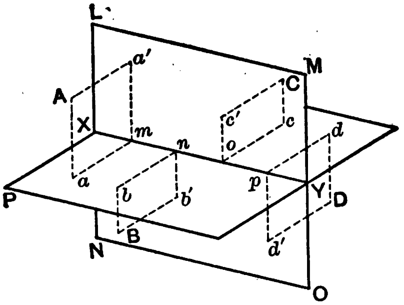


FIG. 2.

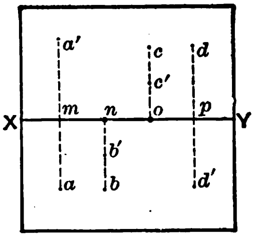


FIG. 3.

The sketch, Fig. 2, represents the vertical and horizontal planes meeting in XY, with points in each of the four quadrants or dihedral angles. The drawing, Fig. 3, represents the orthographic projections of the same points.

In Fig. 2, the point A is situated above the horizontal and in front of the vertical, and its projections are a and a'. These projections are correctly drawn in Fig. 3, a representing a projected point in the horizontal in front of the vertical and a' one in the vertical above the horizontal. In Fig. 2, the perpendicular projecting lines for the point A are Aa and Aa', $a'm$ and am are respectively parallel and equal to these; hence am, $a'm$ are both perpendicular to XY, and when the planes are folded into one, they will form one straight line aa' perpendicular to XY, as in Fig. 3. The perpendicular distance of a, Fig. 3, from XY will be the true length of $Aa' = am$, Fig. 2; and the perpendicular distance of a' from XY, Fig. 3, will be the true length of $\mathrm{A}a = a'm$, Fig. 2. The point B, which is below the horizontal and in front of the vertical, is similarly projected in b and b', and since b', the vertical projection, falls below XY, Fig. 2, it is represented below XY in Fig. 3. b is then the plan and b' the elevation of the point B in the given position, and the distances in Fig. 3 are equal to the true lengths of the similarly lettered lines in Fig. 2. The point C is situated above the horizontal but behind the vertical. Its projections are similarly found in c and c', and in the drawing (Fig. 3) the former point (the plan) is represented above XY because it is behind the vertical. The point D is in the third quadrant (i.e., below the horizontal and behind the vertical). Its plan d falls above XY and its elevation d' below XY.

Hence, to find the projections, first determine the plan and through it draw a line perpendicular to XY. The elevation will be found on this line. When a point is in front of the vertical and above the horizontal, i.e., in the first dihedral angle, its plan is below and its elevation above XY. When it is in front of the vertical and below the horizontal, i.e., in the second dihedral angle, its plan is below and its elevation is also below XY. When it is below the horizontal and behind the vertical, i.e., in the third dihedral angle, its plan is above and its elevation below XY. When it is above the horizontal and behind the vertical, i.e., in the fourth dihedral angle, its plan and its elevation are both above XY.

Observe that the line joining the plan and elevation of the same point is always perpendicular to XY. (See Fig. 3.)

Exercises.—Find the projections of points situated as follows:

A, 1″.5 above H.P. and 2″.5 in front of V.P.
B, 1″.0 below H.P. and 2″.0 in front of V.P.
C, 1″.5 above H.P. and 0″.5 behind V.P.
D, 2″.5 below H.P. and 1″.0 behind V.P.
E, in H.P. and 2″.0 behind V.P.
F, in V.P. and 2″.0 above H.P.
G, in H.P. and 2″.0 in front of V.P.
H, in V.P. and 2″.0 below H.P.

The projections of each of these points are represented by the corresponding small letters in Fig. 4.

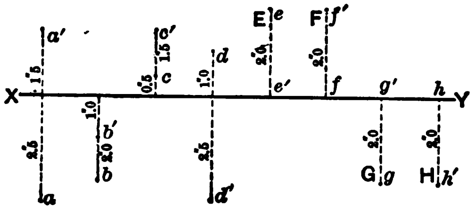


FIG. 4.

PROBLEM II.—Given the projections of a point, to determine its position with reference to the projection planes.

This is precisely the converse of the previous problem. The perpendicular distances of the projections from XY are the distances of the point from the planes, and the general rules given in Problem I may also be employed to determine the position of any given point from its projections: In Fig. 3, the plan *a* is below XY and the point is, therefore, in front of the vertical and at a distance *am* from it. The elevation *a′* is above XY and hence the point is above the horizontal and at the distance *a′m* from it.

Similarly the point B, which has its plan below XY and its elevation also below it, is situated in front of the vertical and below the horizontal. The point C, which has both its plan and elevation above XY, is situated behind the vertical and above the horizontal. The point D, which has its plan above and its elevation below XY, is situated behind the vertical and below the horizontal.

Exercise.—Define the positions of A, B, C, D, E, F, as represented by the projections in Fig. 5.

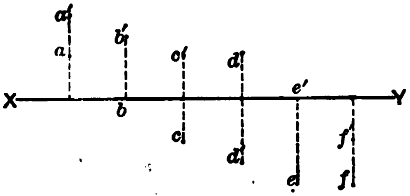


Fig. 5.

Problem III.—Given the position of a line with reference to the projection planes, to determine its projections. [The projection of a line on a plane contains the projections of all the points in the line, and the projection of a straight line is a straight line.]

For convenience a line of given length will be employed. When the positions of the ends of the line are known and the projections of these found by the method of Problem I, the required projections are known.

Example 1.—A line AB 0″.7 long, perpendicular to the horizontal and 0″.35 distant from the vertical. (See Fig. 6.)

Draw a line perpendicular to XY, measure downwards from XY the distance 0″.35 to *a*, which will be the plan of the line, since it is perpendicular to the projection plane. If one end A of the line be taken in the H.P., the vertical projection of A will be *a′* in XY. Measure on the upper side of XY a length *a′b′* of 0″.7 This is the required elevation. The elevation has the same length as the line because it is parallel to it. Here the object is taken as being in the first dihedral angle. It may also be represented in each of the other dihedral angles.

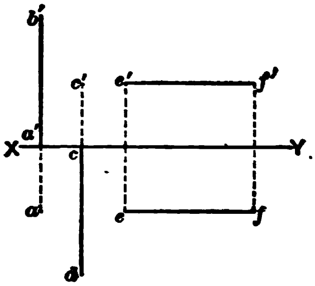


Fig. 6.

Example 2.—A line CD, 0″.7 long, perpendicular to the vertical and 0″.35 distant from the horizontal. (Fig. 6.)

Proceed similarly to the last example, but now fix the vertical projection first. As represented in Fig. 6, the line is in the first quadrant with one end in the vertical plane.

Example 3.—A line EF, 0″.7 long, parallel to both planes and 0″.35 distant from them. (Fig. 6.)

Take any point *e*, 0″.35 from XY and below it. Through *e* draw *ee′* perpendicular to XY, *e′* being 0″.35 above XY. Through *e* and *e′* draw the plan *ef* and the elevation *e′f′*, both 0″.7 long; *ff′* is perpendicular to XY. Here both plan and elevation have the same length as the line, because they are both parallel to it. The line as represented in Fig. 6 is in the first quadrant.

Consider as an exercise how the projections would be affected if the line were similarly placed in other quadrants.

Example 4.—A line parallel to one plane, but inclined at a given angle to the other. (Fig. 7.)

Let the line AB be $0''.7$ in length and inclined at 60° to the horizontal. Let the end A be in the horizontal at a, Fig. 7. The vertical projection of A is a' in XY. Since the line is parallel to the vertical, the vertical projection is also inclined at 60° to the horizontal, and therefore to XY. Draw $a'b'$ at 60° and $0''.7$ long. Drop a perpendicular to XY from b' and through a draw a line parallel to XY to meet the perpendicular in b; ab is the required plan. $a'b'$ is equal in length to AB since it is parallel to it, but the plan ab is foreshortened.

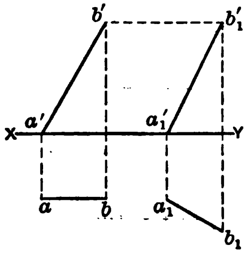


Fig. 7.

Find as an exercise the projection of a line $1''.5$ long when parallel to the horizontal and at 30° to the vertical plane.

Example 5.—A line inclined to both planes and having its projection on one plane at a given angle to the other projection plane.

Let the line be as in the last exercise, at 60° to the H.P., but with the horizontal projection inclined at 30° to the V.P., i.e., to XY.

Make first the same construction as for Ex. 4. Then draw a line a_1b_1 equal in length to ab, but at the required angle of 30° to XY. This is the required plan, because the angle of inclination to the horizontal being the same as in the last problem, the length of the plan remains the same, although its position is changed. Now, since the inclination to the horizontal remains the same, the point B is at the same distance above the plane and its new elevation is, therefore, in the line drawn through b' parallel to XY. Since it is also in the perpendicular to XY drawn through b_1, it is at b'_1. The elevation of a_1, since it is in the horizontal, is at a'_1. Hence the required elevation is $a'_1b'_1$. In this case, since the line is not parallel to either plane, both projections are foreshortened. If the plan a_1b_1 were at a larger angle to XY, the elevation $a'_1b'_1$ would be still further foreshortened. When the plan a_1b_1 is placed at a right angle to XY, the elevation $a'_1b'_1$ is also found to make a right angle with XY, and in this case the line A_1B_1, which makes an angle of 60° with the horizontal, will make 30° with the vertical plane.

Do not confuse the angle between the *plan* of the line and XY (or the V.P.) with the angle between the line itself and the vertical plane.

Another method may be employed to obtain the required elevation. (Fig. 8.) Imagine the vertical plane to make the required angle of (say) 45° with its former position on the horizontal, i.e., take a new intersection line X_1Y_1 (Fig. 8) making 45° with ab and the former XY. Erect perpendiculars aa'_1, bb'_1 and on the latter measure off from X_1Y_1 an altitude equal to the altitude of b' above XY. ab is the plan and $a'_1b'_1$ the elevation on the plane meeting the horizontal in X_1Y_1.

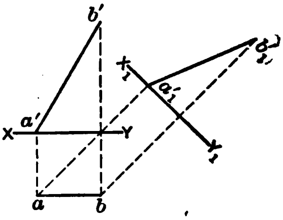


FIG. 8

Find, as an exercise, by the methods of Fig. 7 and Fig. 8, the projections of a line 1″.5 long when it makes an angle of 45° with the horizontal and its plan is at 60° to XY.

Find also the projections of the same line when it makes an angle of 60° with the vertical and its elevation makes an angle of 45° with XY.

Example. 6.—A line inclined at any given angle to both planes of projection. This is the general case. (Fig. 9.)

Let the angle which the given line AB makes with the vertical be 30° and that with the horizontal 40°. Take any point *a* in one plane, say the horizontal, and find its elevation *a'*. From *a* draw a line *ao*, making with XY an angle of 30°—the required angle with the vertical—and from *a'* draw a line making with XY an angle of 40°—the required angle with the horizontal. Make these lines *ao*, *a'n* equal to the length of the given line AB, and from the points *o* and *n* drop perpendiculars on XY, meeting it in *p* and *q*; *a'p* is the length of the projection of the line AB on the V.P., and *a'q* the length of its projection on the H.P. From *a* with radius equal to *a'q* describe a circle. The extremity B of the line AB must have its horizontal projection in this circle. Similarly from *a'* describe a circle with radius *a'p*, the length of the projection on the vertical plane. The projection of B on the V.P. must lie in this circle, but it must also lie in the horizontal line *nb'* drawn through *n*, since *nq* is necessarily the distance of the point B above the horizontal. The vertical projection of B is therefore at *b'*, and *a'b'* is the required vertical projection. From *b'* drop a perpendicular to meet the lower circle in *b* and join *ab*, which is the required plan. The point *b* may also be found by drawing a line through *o* parallel to XY to meet the circle. There are obviously four positions above the horizontal in which the line may be drawn to satisfy the conditions of the problem. Find the other three. The essential point of this problem is to determine the lengths of the projections, and this may be done anywhere on the paper.

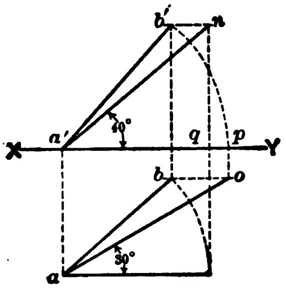


Fig. 9.

The precise arrangement of the drawing as given in Fig. 9 is not important.

Exercise.—Find the projection of a line 1″.2 long which makes an angle of 30° with the H.P., and an angle of 45° with the V.P., (*a*) when one end of it is in the H.P., (*b*) when one end of it is in the V.P., (*c*) when one end of it is 0″.5 distant from both planes.

Problem IV.—Given the projections of a straight line, to determine its length and its inclination to the projection planes. (Fig. 10.)

Let AB be the straight line projected in *ab*, *a′b′*. Swing the line about one end until it is parallel to one of the projection planes, while it still retains its inclination to the other plane. The projection on the plane to which it is made parallel will then be equal to its true length, and the angle between this projection and XY will be its inclination to the other plane.

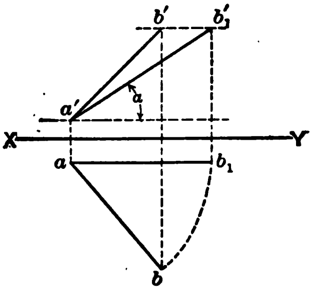


Fig. 10.

Describe the arc bb_1 from a as a centre and draw ab_1 parallel to XY. The altitude of B has not been changed, hence its elevation is found in the intersection of the horizontal line through b' and the perpendicular from b_1. Join this intersection point b'_1 to a'; b'_1a' is the vertical projection of BA when the latter is parallel to the V.P. It is, therefore, equal to BA in length. The angle α which b'_1a' makes with XY is equal to the inclination of BA to the H.P.

To find the inclination to the V.P., the vertical projection should in a similar manner be brought parallel to XY, i.e., to the H.P., by swinging it about A, while the line still retains its inclination to the V.P. This will serve also to measure the true length of the line. (The line might with equal facility, in both cases, have been swung about the point B.)

Take, as an exercise, projections similar to those in Fig. 10, page 11, and make the measurements by swinging first about A and then about B.

Another Method. (Fig. 11.)—Let the point A be situated at a distance $a'm$ above a, and B at a height $b'n$ above b. If perpendiculars aA_1, bB_1 equal respectively to $a'm$ and $b'n$ be drawn to ab, the line A_1B_1 will be equal to AB, and the inclination (α) to ab will be the angle which AB makes with the horizontal. Similarly, if perpendiculars be drawn at a' and b' to the line $a'b'$ equal to the perpendiculars from A and B on the V.P., that is, to am and bn, the line A_2B_2 joining the extremities of these perpendiculars will be equal to AB, and the angle (β) between A_2B_2 and $a'b'$ will be the inclination of AB to the vertical.

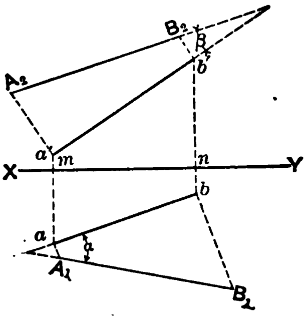


Fig. 11.

Problem V.—To find the traces of a line, i.e., the points where the line would, if produced, meet the projection planes. (Fig. 12.)

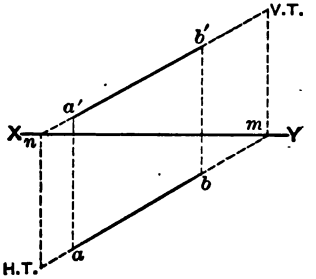


Fig. 12.

Let AB be the line; produce one of the projections, as ab, until it meets XY. At the point of meeting m, erect a perpendicular to meet the production of the other projection $a'b'$ in V.T., which is the trace in the vertical plane. Similarly, produce $a'b'$ to n in XY, and erect a perpendicular to meet ba produced in H.T., which is the horizontal trace. In

Fig. 13 are represented cases where the portion of the line between the traces does not lie in the first quadrant as above.

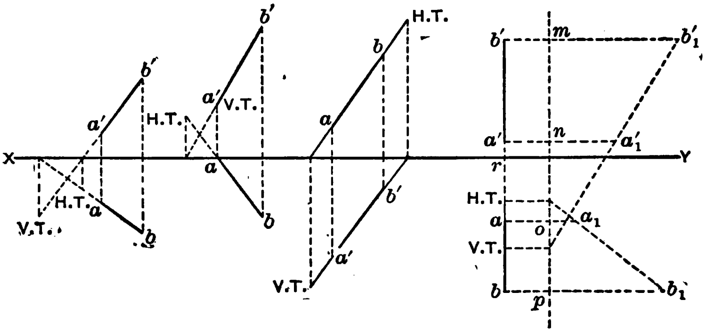


Fig. 13.

The same method, however, applies to all cases. The H.T. will, of course, always lie in the horizontal projection or its production, and the V.T. in the vertical projection or its production.

The figure to the right in (13) represents the special case in which the projections are in a plane at right angles to both projection planes. A special construction is here necessary. Draw a line *mnop* perpendicular to XY, representing a plane perpendicular to both projection planes, and measure $mb'_1 = rb$ and $na'_1 = ra$; $b'_1a'_1$ may be considered a plan on the perpendicular plane corresponding to the given elevation $a'b'$. From this new-found plan and the former elevation the V.T. is found as in former examples. Similarly by finding an elevation in a_1b_1 to correspond with the given plan ab the H.T. is found.

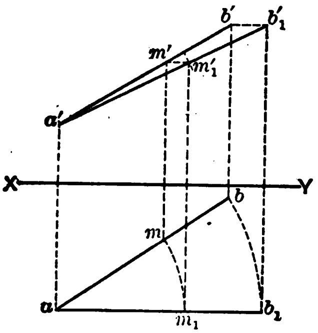


Fig. 14.

PROBLEM VI.—Given the projections of a straight line, to mark off a given distance along the line. (Fig. 14.)

This is an application of the method of Problem IV. Let AB be the given line; it is required to mark off a length equal to the line D, from A. Swing the line about A until it is brought parallel to the vertical in the projections ab_1 and $a'b'_1$. From a' mark off the length $a'm'_1$ equal to D and find the plan of m'_1 in m_1. Now return AB to its original position; m_1 is transferred to m by an arc from a as centre, and m'_1 to m' by a horizontal line drawn through m'_1. mm' should be perpendicular to XY. AM, of which am and $a'm'$ are the projections, is the required length.

Prove the result of this method first by keeping B fixed and swinging the plan; second, by keeping A fixed and swinging the elevation; third, by keeping B fixed and swinging the elevation.

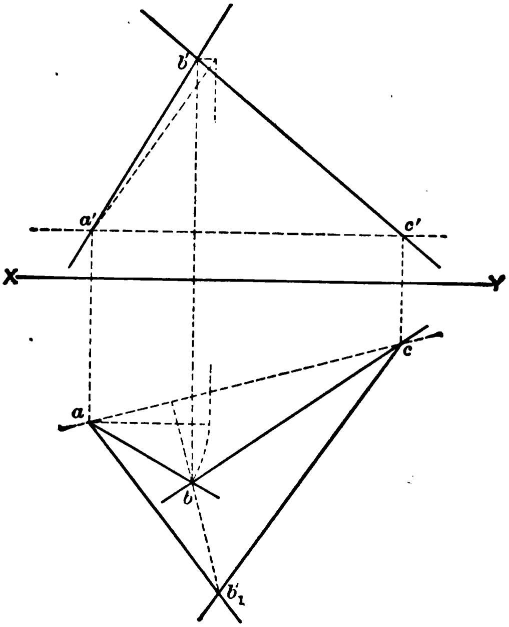


FIG. 15.

PROBLEM VII.—Given the projections of two straight lines which intersect, to find the angles between them. (Fig. 15.)

Let AB, BC be the lines and let the line AC be horizontal, i.e., draw the vertical projection line *a′c′* parallel to XY and find the plan of the points A and C by drawing the lines *a′a*, *c′c*, perpendicular to XY. Find the true lengths of the lines AB and BC, as in Problem IV, and on *ac* as a base describe the triangle acb_1, having the sides ab_1, and cb_1 respectively equal to AB and BC; bb_1 will be found perpendicular to *ac*. The angles at b_1 will be equal to the required angles at B between AB and BC. The points A and C might have been taken in the H.P., or a similar construction might have been made by taking points in the V.P. or in a plane parallel to it.

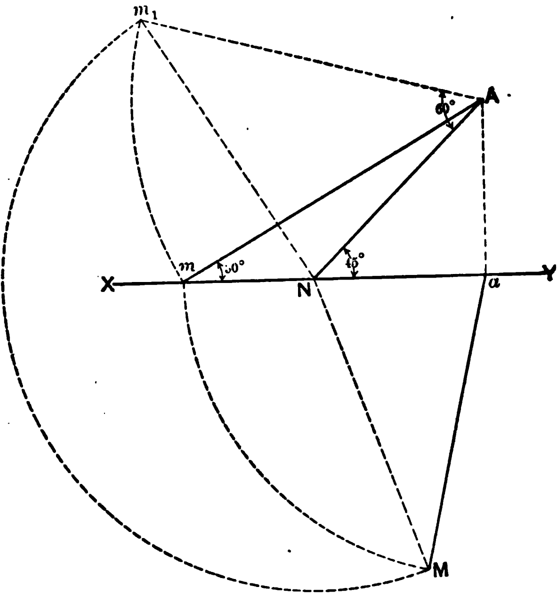


FIG. 16.

PROBLEM VIII.—To find the projection of a given angle when the sides containing it make given angles with the projection plane. (Fig. 16.)

Let it be required to find the horizontal projection of an angle of 60° when the sides containing it make angles of 30° and 45° respectively with the H.P.

Take any point A in the V.P. and from it draw lines meeting XY in m and N, and making with XY the given angles 30° and 45°. The lines Am and AN are now to be made to contain the angle of 60°. Let one of them, AN, be retained in the V.P. while the other, Am, is swung about the vertical line Aa, the point m describing a circle in the H.P. about a as a centre until it makes with AN the angle of 60°. In order to measure this angle it is necessary to construct a triangle having sides equal to AN, Am, and containing the given angle of 60°. Let such a triangle be constructed in NAm_1. Nm_1 equals the required side in the H.P. which will subtend the angle of 60°. Hence from N as centre describe an arc with radius Nm_1 to meet the arc described from a with radius am in M. NaM will be the required projection of the given angle.

EXERCISES ON CHAPTER I.

1. The point A is 1″ above the horizontal and 1″.5 in front of the vertical. The point B is 0″.5 above the horizontal and 0″.5 in front of the vertical. The projecting planes containing the points are 1″ apart. (*a*) Represent the projections of the points. (*b*) Find the distance between the points. (*c*) Find the angles which AB makes with the projection planes. (*d*) Find the traces of the line. (*e*) Find the true distance between the traces.

2. ab, the plan of a line, makes an angle of 30° with XY and is 2″ in length. The point B is in the H.P., and the point A is in the V.P. 1″.5 above the H.P. (*a*) Find the plan and elevation of the line. (*b*) Find the length of the line and the angles which it makes with the projection planes.

3. The elevation *a'b'* of a line is inclined at 45° to XY and is 2″ long. *a'* is 1″ above XY and *b'* is also above XY. A is ½″ in front and B 2″ behind the V.P. Find the plan and elevation of the line and the distance between its traces.

4. A horizontal line 3″ long is 1½″ above the H.P. One end of the line is in the V.P. and the other end is 1″ in front of it. Find the projections of the line.

5. What is the true length of a line the plan of which measures 1″.5 when the line is inclined at 25° to the horizontal?

6. A line 3″ long has a plan 1″.5 in length. Find the inclination of the line to the horizontal.

7. Find the projections of a line which is inclined at 45° to the horizontal and 30° to the vertical, the lower end of which is at a point 0″.5 from both planes of projection. Represent the four positions of the line which fulfil these conditions.

8. The horizontal projection *abc* of a triangle ABC is an equilateral triangle, and *ac* is inclined to XY at 45°; *a'c'* is horizontal and *a'b'* is inclined to XY at 30°. Find the angles of the triangle.

9. Find the projection of an angle of 75° when the sides containing it make angles of 15° and 30° with the H.P.; (*a*) when the side inclined at 15° is in the V.P.; (*b*) when the side inclined at 30° is in the V.P.

10. Find the plan of a line 2″ long, when the line is inclined at 30° to the H.P., and both projections are perpendicular to XY. The end of the line nearest XY is 0″.5 above the H.P. and 0″.25 in front of the V.P. (*a*) Find the traces of the line.

CHAPTER II.

LINES AND PLANES.

A PLANE is a surface such that any two points being taken in it the straight line joining them lies wholly in the surface. A plane may be represented by its *traces* or, when of finite dimensions, by its projections. The *traces* of a plane are the straight lines in which the plane meets the projection planes.

PROBLEM I.—Given the position of a plane figure, to determine its projections.

This is merely an extension of Problem III, Chapter I, in which it was required to determine the projections of a line, its position being given.

Example 1.—A rectangle ABCD, when its plane is perpendicular to and one edge parallel with the V.P. (Fig. 17.)

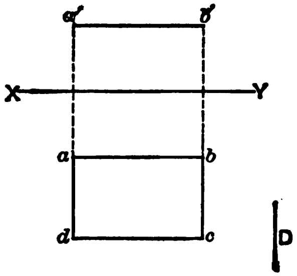


FIG. 17.

Let the rectangle be at the distance D above the horizontal, and the nearer edge the same distance from the vertical, and let the longer edge AB be parallel to the vertical. Draw *ab*, *a'b'*, representing the plan and elevation of AB at the distance D from XY. Since the rectangular plane is parallel to the horizontal, its plan is equal to it in every respect. The rectangle *abcd* constructed upon the side *ab* is, therefore, the required plan, and since the plane is perpendicular to the vertical, its elevation is the line $a'b'$. This line might also be described as $d'c'$, since it is the elevation of DC as well as AB.

Example 2.—A rectangle ABCD, when it is perpendicular to both projection planes. (Fig. 18.)

Here the projections on both planes are lines. Let the shorter edge be parallel to the horizontal. Then the plan of the plane is a line ab, equal in length to the side AB, and perpendicular to XY, and the elevation is a line $a'd'$ equal in length to AD. The plane is represented as having its shorter edge in the horizontal.

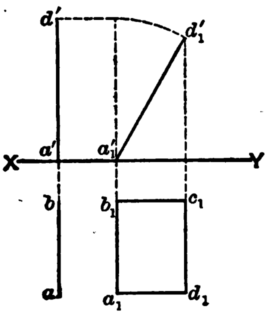


FIG. 18.

Example 3.—A rectangle ABCD, when it is perpendicular to the vertical, but inclined to the horizontal at 60°. (Fig. 18.)

The required projections may conveniently be derived from those in the previous example. Imagine the plane to be turned about the edge ab until it is inclined at the required angle. The angle between the plane and the horizontal is the same as the angle between the elevation (the line $a'_1d'_1$) and XY, since the plane remains perpendicular to the vertical. Let the elevation, therefore, be turned about a'_1 until it makes an angle of 60° with XY, and from d'_1 drop a perpendicular meeting the lines drawn from b_1 and a_1 parallel to XY in c_1 and d_1. The rectangle $a_1b_1c_1d_1$ is the required plan. The lines AB and CD, being parallel to the horizontal, have projections a_1b_1 and c_1d_1 equal to them, but AD and BC, being inclined to the horizontal, have foreshortened plans. The plane, being still perpendicular to the vertical, is represented there by a line equal in length to the sides which are parallel to that plane.

Example 4.—A rectangle ABCD, making an angle of 60° with the horizontal, when one edge is parallel to the horizontal, but at an angle of 45° with XY. (Fig. 19.)

Let the shorter edge be parallel to the horizontal. First draw the projections when the plane is inclined at 60° as required, but perpendicular to the V.P., as in Example 3. The plan *abcd* thus obtained is of the form required, but its position with reference to XY is to be changed so that *ab* shall make the required angle of 45° with XY. Copy the plan as in the second part of the figure: the plan a_1b_1 of the horizontal edge AB being at the required angle of 45° with XY. The former elevation $a'd'$ now gives the altitudes of the several angles, and from it and the plan $a_1b_1c_1d_1$ in its proper position the required elevation is obtained. Draw perpendiculars to XY from a_1, b_1, c_1, and d_1 to meet horizontals from a' and d'; $a'_1b'_1c'_1d'_1$ is the required vertical projection.

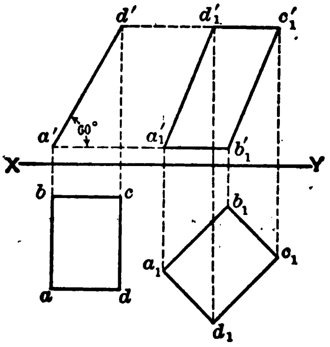


FIG. 19.

Another Method.—The projection may be more quickly found by the second method of Ex. 5, Prob. III, Chap. I. (Fig. 20.)

First find the projections when the plane is perpendicular to the V.P., as in Ex. 3. Then draw X_1Y_1 at 45° to *ab* (the angle which the side AB is required to make with the V.P. or with the line XY), and erect perpendiculars from *a*, *b*, *c*, and *d* to X_1Y_1. Measure the altitude of d' above XY and draw the horizontal line $c'_1d'_1$ at the same distance above X_1Y_1. $a'_1b'_1c'_1d'_1$ is the required vertical projection.

Example 5.—A rectangle ABCD, when one side is inclined at any given angles to the projection planes. (Fig. 21.)

Let AD, one of the longer sides, be inclined at 45° to the horizontal and 30° to the vertical, and let the other side be parallel to the H.P. By the method of Prob. III, Chap. I, Fig. 9, find the projections of AD in *ad* and *a′d′*. A separate construction is given for this in the figure and *ad*, *a′d′* drawn parallel to the projections obtained. Since AB is perpendicular to AD and parallel to the horizontal, its projection *ab* is perpendicular to *ad*. Draw *ab* and

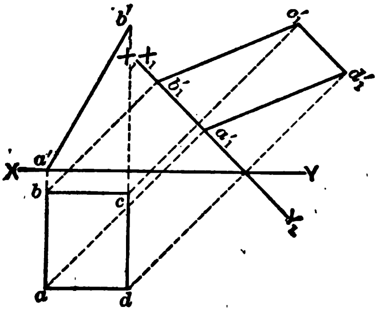


FIG. 20.

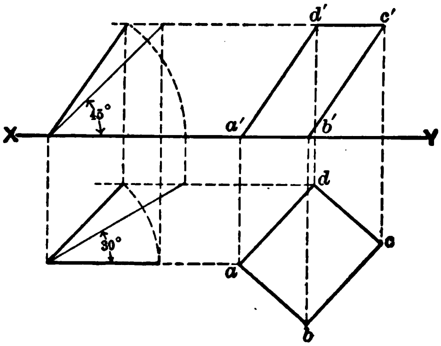


FIG. 21.

make it equal to AB. The completed rectangle *abcd* is the required plan, and since the lines AB, DC are horizontal, their vertical projections are also horizontal. Draw also perpendiculars to XY from *b* and *c* to meet horizontals from *a'* and *d'*, to complete the vertical projection *a'b'c'd'*.

PROBLEM II.—Given the projections of any plane figure ABC, to determine the traces of the plane containing it. (Fig. 22.)

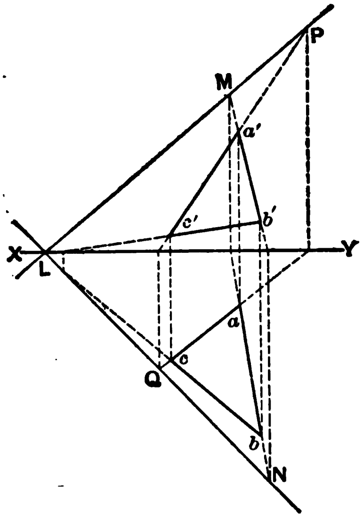


FIG. 22.

By the method of Prob. V, Chap. I, find the vertical trace M and the horizontal trace N of the line AB. Similarly, find the traces P and Q of the line AC. The points N and Q, being both in the plane of ABC and in the H.P., are in the horizontal trace of the plane, and NQL is that trace. For similar reasons, PML is the vertical trace of the plane. If the traces of BC be obtained, they will also be found to lie on the lines NL, PL.

Since, in general, any three planes cutting each other must meet in a point, the horizontal and vertical traces of a plane must always meet on XY.

PROBLEM III.—Given the projections of any plane figure ABC, to determine its true form. (Fig. 23.)

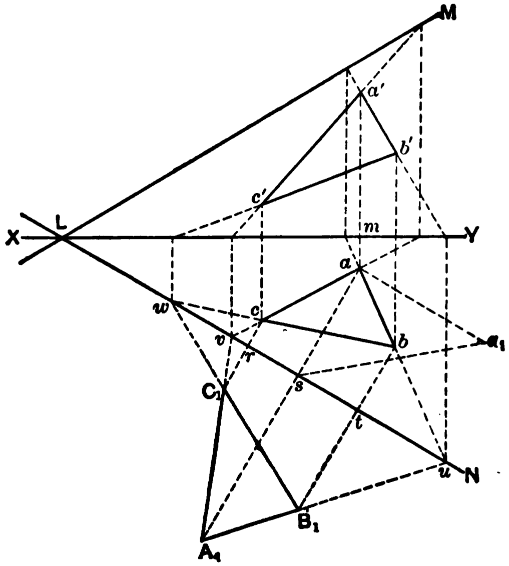


FIG. 23.

This problem may be solved by the method of Prob. IV, Chap. I, by determining the lengths of the sides and of such diagonals as are necessary (in the case of polygons) to fix the figure, and hence constructing the figure. Or it may be determined by finding the traces of the containing plane as in Prob. II, and swinging this plane about one of its traces into the corresponding projection plane. This is called rabatment. Find the horizontal trace LN of the plane containing ABC, and imagine the plane to be swung about LN backwards into the horizontal. As the plane swings around the trace each point in the plane, as A, B, and C, describes a circle about LN, perpendicular to the H.P. Hence the projection of the locus of each point is a straight line perpendicular to LN and the rabatted positions of the points are somewhere in these perpendiculars. Through a, b, and c draw perpendiculars asA_1, crC_1, and btB_1 to LN. The point A is vertically over a, and Aa makes with as a right angle; Aas is, therefore, a right-angled triangle of which the sides as and $\mathrm{A}a = a'm$ are known and the hypotenuse As may be found. Construct the triangle by drawing aa_1 at right angles to as and making $aa_1 = ma'$. sa_1 is the required hypotenuse. Make $s\mathrm{A}_1 = sa_1$. A_1 is the rabatment of A. Now produce ab and ac to meet LN in u and v. As the plane is swung round, these points do not move, and the lines $v\mathrm{A}_1$, $u\mathrm{A}_1$ are the rabatments of the lines Av and Au, and hence the points B and C are in these lines and are at B_1 and C_1, where the perpendiculars from b and c meet the lines. The points B_1 and C_1 might have been found by the same method as A_1. Where the intersection is acute, as in C_1, a second determination of its position may be desirable, and may usually be made by producing another line, such as bc, to a point w in the horizontal trace and joining B_1w, which, being the rabatted position of Bw, also contains C.

The construction may with equal facility be made by rabatting about LM into the vertical.

Problem IV.—Given the projections of two lines which do not meet, to find the traces of the plane containing one of them and being parallel to the other. (Fig. 24.)

A line is parallel to a plane when it is parallel to any line in the plane.

Let AB and MN be the lines. Take any point O in AB, and through o and o' draw lines parallel to mn and $m'n'$; these being the projections of a line parallel to MN and meeting AB in O. Find the traces of this line in S and T and find the traces of AB in P and Q. PT will mark the horizontal trace and SQ the vertical trace of a plane containing AB and being parallel to MN. These traces will be found to meet in XY in L.

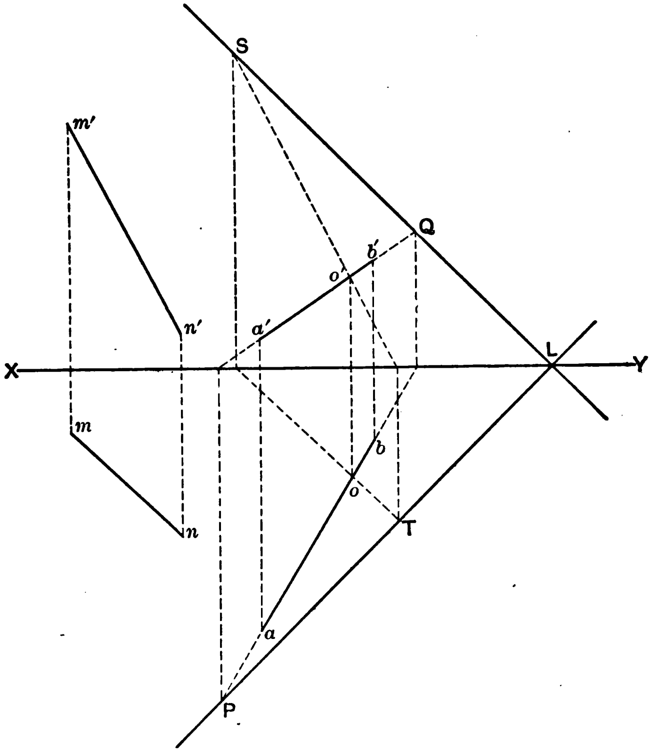


FIG. 24.

PROBLEM V.—To determine a plane which shall be parallel to a given plane and contain a given point. (Fig. 25.)

Planes which are parallel have their traces parallel.

Let P be the point and MLN the given plane. Through P draw a horizontal line PQ parallel to LN, the horizontal trace of the plane. The line PQ will be parallel to the given plane since it is parallel to a line LN in it, and will therefore be a line in the required plane. Its trace Q is in the vertical trace of the required plane. Through Q draw QR parallel to ML, and through R draw RS parallel to LN. QRS will be the traces of the required plane.

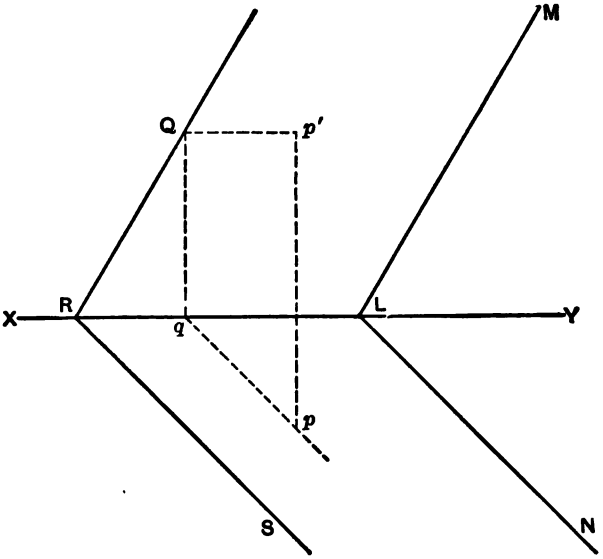


FIG. 25.

The required plane may of course be found by drawing through the given point a line parallel to *any* assumed line in the plane and drawing through the traces of the parallel line thus found the traces of the required parallel plane. It is convenient to use LN as above, or ML, for this purpose.

PROBLEM VI.—To find the point in which a given line meets a given plane. (Fig. 26.)

Let MLN be the given plane and AB the given line. The method is to take any plane containing the line and find the line in which this plane meets the given plane.

The point of meeting of this found line and the given line will be the required point. A line will be contained by any plane the traces of which contain the traces of the line, and the line of intersection of any two planes is the line joining the points of meeting of the traces of the two planes in the H.P. and V.P. respectively, which points are also the traces of the line of intersection. In special cases, where the traces are all parallel or all converge to a common point in the XY line, a third plane will be required to find points in the line of intersection of two planes.

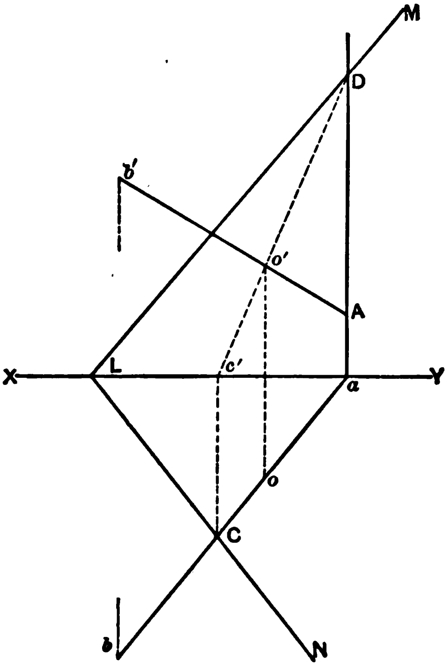


FIG. 26.

For convenience take a vertical plane containing the line. The horizontal trace of this plane will coincide with the plan of the line *ba*. Draw the vertical trace *a*D. Now find the line of intersection of the two planes which meet in the H.P. in C and in the V.P. in D. The plan of CD will be C*a* and its elevation *c*′D, and this line will be met by the given line in O which is the required point.

PROBLEM VII.—To measure the perpendicular distance between a point and a plane. (Fig. 27.)

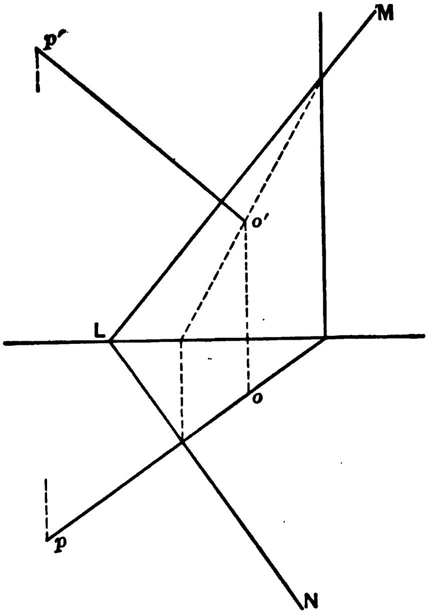


FIG. 27.

Let P be the given point and MLN the given plane. From *p* and *p*′ drop perpendiculars on the traces of the plane MLN. These will represent the projections of a line perpendicular to the plane. (When a line is perpendicular to a plane the traces of the plane are perpendicular to the projections of the line.) Find the point of meeting between the line and the plane (Prob. VI, Fig. 26) in O. The length of PO is the required distance and is to be found by the method of Prob. IV, Chap. I, Fig. 10.

Cor.—If it be required to find a plane at a given distance from the given plane, this distance is to be measured from O along OP and a plane drawn through the point thus found parallel to the given plane (Prob. V, Fig. 25).

Problem VIII.—To find a plane which shall be perpendicular to a given line and pass through a given point. (Fig. 28.)

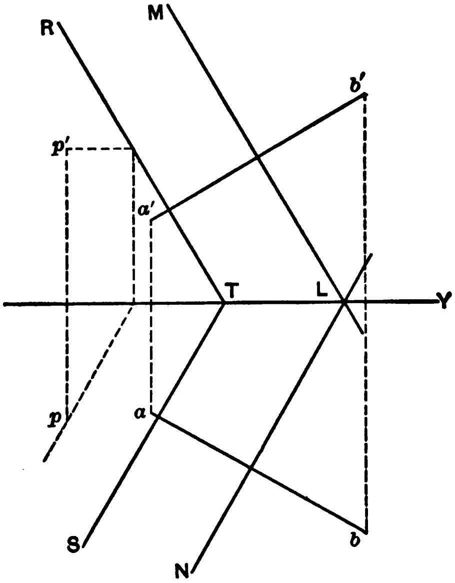


Fig. 28.

Let AB be the given line and P the given point. Draw any plane MLN perpendicular to the given line by drawing traces perpendicular to the projections of the line. Through the point P draw the required plane RTS (Prob. V, Fig. 25) parallel to the plane MLN.

Cor.—If it be required to find the perpendicular from P on the line AB, determine the point of meeting of the line AB with the plane RTS (Prob. VI, Fig. 26) and join this point to the point P. This will be the required perpendicular.

Problem IX.—Given the traces of any plane to find the angles which the plane makes with the projection planes. (Fig. 29.)

[The angle between any two planes is the angle between the intersection lines caused by a plane drawn at right angles to the line of meeting of the two planes. This is called the *dihedral* angle.]

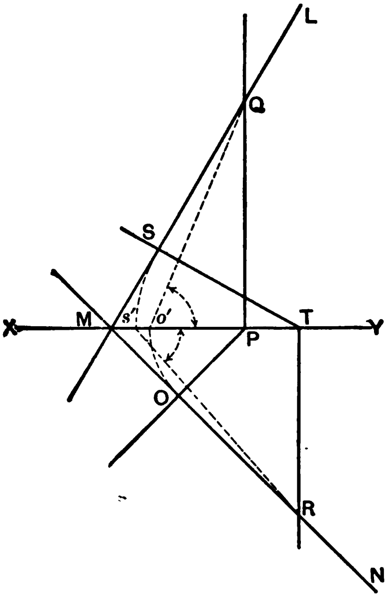


Fig. 29.

Let LMN be the traces of the plane. Draw OP perpendicular to MN, and PQ perpendicular to XY. OPQ represents a plane perpendicular to NM and therefore to both the H.P. and the given plane LMN. Construct the right-angled triangle *o*′PQ by swinging OP on to XY. Q*o*′P is the inclination of LMN to the H.P., and similarly the inclination to the V.P. is found in R*s*′T.

Problem X.—To find a plane which shall make given angles with the projection planes. (Fig. 30.)

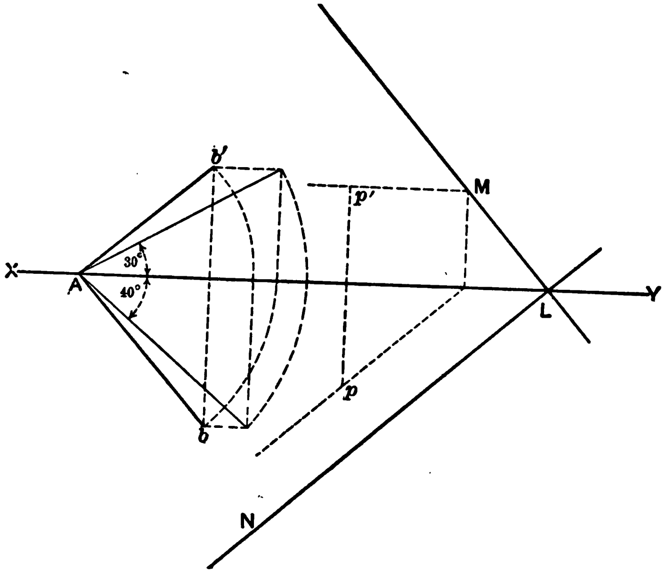


Fig. 30.

Take the complements of the given angles and project a line (Prob. III, Chap. I, Fig. 9) to make these complemental angles with the projection planes. The required plane will be perpendicular to the line so found, and traces drawn at right angles to the projections of the line will represent the required plane. If it be required to draw the plane through a given point, a plane parallel to that found as above may be so drawn by Prob. V, Fig. 25.

Let a plane be required which shall make angles of 60° and 50° with H.P. and V.P. respectively, *and pass through* a given point P. Draw a line AB to make an angle of 30° with the H.P. and an angle of 40° with the V.P. Through P draw a horizontal line perpendicular to AB, and through its trace M draw the vertical trace of the required plane, which will be perpendicular to Ab', and through L draw the horizontal trace LN perpendicular to Ab.

PROBLEM XI.—Given the rabatted position of a point and the traces of a plane, to find the projections of the point when it is raised into the plane. (Fig. 31.)

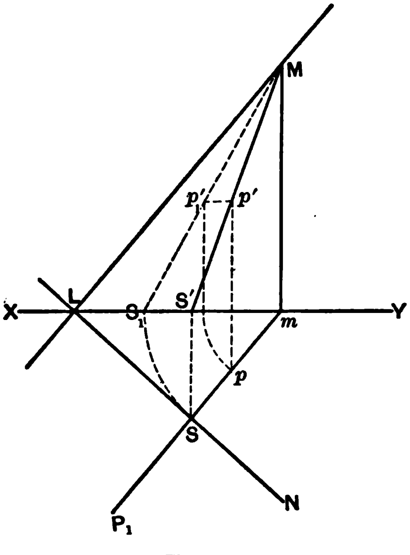


FIG. 31.

Let the point P_1 be in the H.P., and let MLN be the plane into which it is to be raised. Through P_1 pass the vertical plane P_1mM at right angles to LN. As P_1 revolves about LN it will lie in this vertical plane and the locus of its horizontal projection will be P_1Sm. The vertical plane will meet the given plane in SM, which is the hypotenuse of a right-angled triangle with sides Sm and Mm. Swing this triangle about Mm into the vertical and make $S_1p'_1$ equal to SP$_1$. Drop a perpendicular on XY from p'_1 and transfer the point so found to p, which will be the plan of the point in the plane. The vertical projection p' will be found by drawing a horizontal line from p'_1 to meet a perpendicular line from p. p' must also lie in S′M, the vertical projection of the line of intersection of the two planes of which Sm is the horizontal projection.

A similar construction might have been made by rabatting the triangle about Sm into the H.P. and measuring a distance along the hypotenuse from S equal to SP$_1$, as sa_1 in Fig. 23.

Problem XII.—To measure the angles between any two intersecting planes. (Fig. 32.) (Prob. IX, where one of the planes is a projection plane, is a special case of this.)

Let LMN, LON be the two planes intersecting in NL. Draw *rst* to represent the horizontal trace of a plane cutting NL at right angles. It is required to find the length of the perpendicular from *s* on NL the line of meeting of the two planes, in order to determine the intersection lines between the cutting plane of which *rst* is the trace and the given planes. Rabat the vertical plane containing NL into the V.P. about L*l*. NL will be represented by N$_1$L, and *s* by s_1. Drop a perpendicular s_1k_1 from s_1 on N$_1$L. Now rabat the cutting-plane about *tr* into the horizontal, making sk_2 equal to s_1k_1. Join k_2t and k_2r. The required angles are tk_2r and its exterior angle.

Cor. I.—To divide the angles between two planes into any number of parts. For example, to bisect the angles between the given planes.—Bisect the angle tk_2r and its supplement, and produce the bisecting lines to u and v. The traces NRL and vN–LQ represent the bisecting planes.

PROBLEM XIII.—Given a plane and a line in the plane, to draw a plane through the line to make a given angle with the given plane. (Fig. 32.)

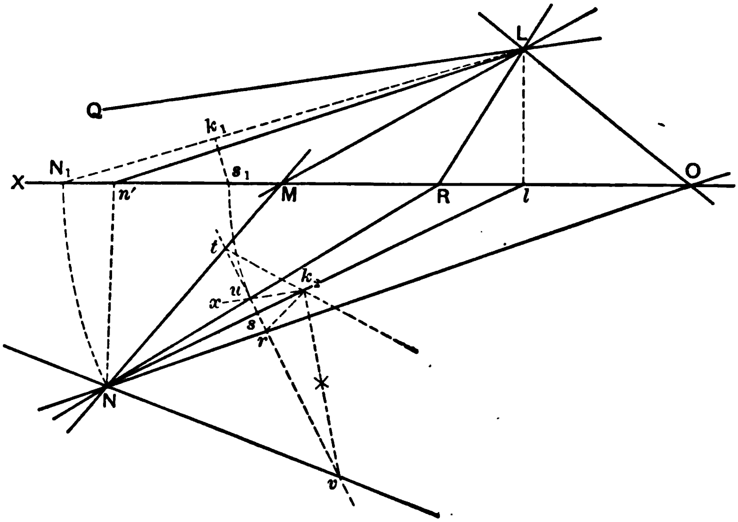


FIG. 32.

Let LMN be the given plane, and LN a line in it. As in Prob. XII, draw rst to represent the horizontal trace of a plane perpendicular to LN. Rabat this line into the vertical and find the length of the perpendicular on it from s, viz., s_1k_1. Make sk_2 equal to s_1k_1 and join tk_2, representing a rabatment of the line of section between the perpendicular cutting plane and the given plane. At k_2 make tk_2r equal to the angle which the required plane is to make with the given plane. Join Nr and produce it to form the horizontal trace NO of the required plane NOL.

PROBLEM XIV.—To find the angle between a given line and a given plane, i.e., the angle between the line and its projection on the given inclined plane. (Fig. 33.)

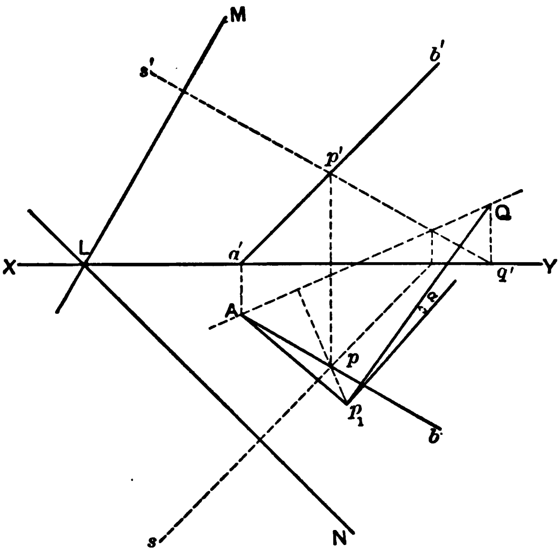


FIG. 33.

Let AB be the given line and MLN the given plane. Take any point P in the line and from it drop a perpendicular PS on the plane. Now find the angle between AB and PS by the method of Prob. VII, Chap. I. This angle Ap_1Q is the complement of the angle between the line and the plane, and $\alpha = (90° - Ap_1Q)$ is therefore the required angle.

PROBLEM XV.—To draw a line from a given point to make a given angle with a given line. (Fig. 34.)

Let AB be the given line and P the given point. Find by Prob. IV, Chap. II, a trace LN of the plane containing the line and the point. By Prob. III, Chap. II, rabat the

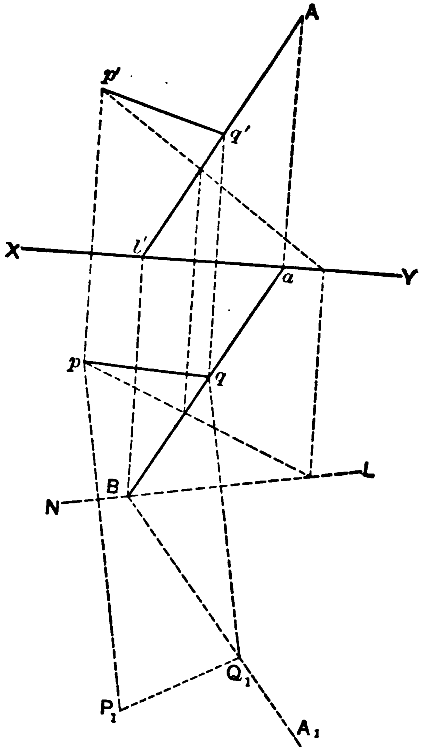


FIG. 34.

point P to P_1 and the line AB into the H.P. in BA_1. Through P_1 draw a line P_1Q_1 to make the required angle with BA_1. By Prob. XI, Chap. II, find the projections corresponding to Q_1 in q' and q. Join pq and $p'q'$ giving the projections of PQ, which is the required line.

PROBLEM XVI.—To find the projection of any figure of which the plane of the figure and one of its edges are inclined at given angles to the projection plane. (Fig. 35.)

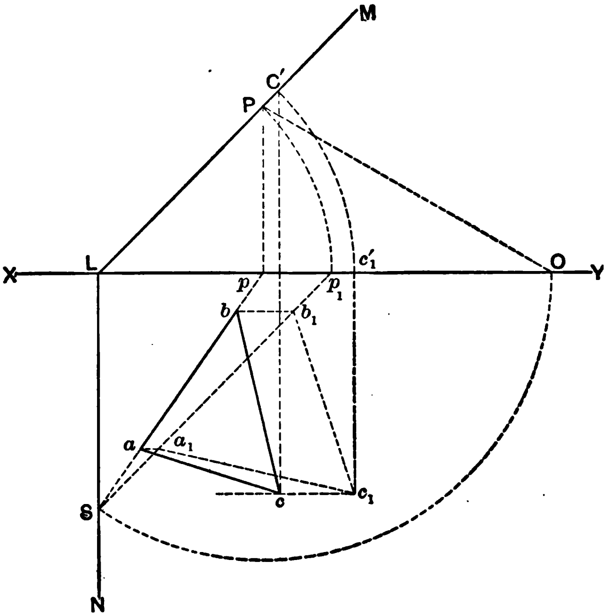


Fig. 35.

Let the figure be an equilateral triangle and let its plane be perpendicular to the V.P. Take the plane MLN at the required angle to the H.P. Take any point P in ML and from it draw a line PO, making the angle with XY which one side of the triangle is to make with the H.P. Make *p*S equal to *p*O. *p*S will represent the plan of a line in the plane and at the required angle to the H.P. Rabat this line into the H.P. in p_1S, and on it construct the equilateral triangle $a_1b_1c_1$. From this find the plan *abc* of the triangle.

If the plane of the figure is also to be inclined to the V.P., a new position of the XY line may now be taken and an elevation derived from the plan *abc* and the corresponding elevations of the points; or, the plane MLN may at once be put in the required position with reference to the projection planes, and the position of the line being determined as above, it may be rabatted and the plan and elevation found as in Prob. XI. The position of XY with reference to the traces is to be determined by Prob. X, when the inclinations of the plane to both planes of projection are given.

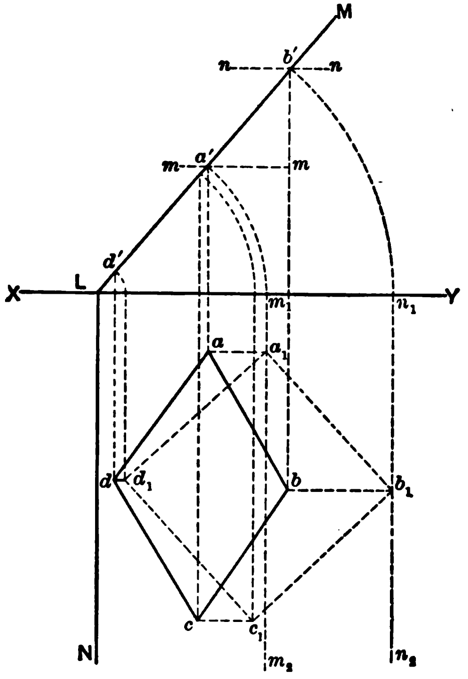


FIG. 36.

PROBLEM XVII.—To find the projections of any plane figure when the altitudes of two points in it and the inclination of its plane are given. (Fig. 36.)

Let the plane MLN be perpendicular to the V.P. and inclined to the H.P. at the given angle. Let m and n represent the altitudes of the angles of a square which it is required to project. If m and n are contained by horizontal planes, these planes will intersect the plane MLN along lines parallel to LN. Rabat these into the H.P. in m_1m_2 and n_1n_2. Take any point a_1 in one of these lines and lay off a_1b_1 so that b_1 shall be in the other line. Construct the square $a_1b_1c_1d_1$ and find its plan *abcd* when put into the plane MLN.

The elevation on any vertical plane may be found by making the necessary change in the position of XY.

PROBLEM XVIII.—To find the projections of any plane figure when the altitudes of any three points in it are given. (Fig. 37.)

Let the altitudes of three adjacent angles, A, B, C, of a pentagon be m, n, and o, respectively. Construct the pentagon A′B′C′D′E′ and mark off the distances AB = A′B′ and Ac'_1 = A′C′, and, the inclinations of the lines AB and AC being thus obtained, find the projection of the angle CAB as in Prob. VIII. Now rabat AB (Prob. III) into the H.P. in A_1B_1 and on it construct the pentagon $A_1B_1C_1D_1E_1 \cdot C_1$ will correspond with the rabatted position of C. From the rabatted figure find the horizontal projection *abcde*. The vertical projections of A, B, and C are known and the vertical projections of the remaining points may be readily determined from the plan and horizontal trace, as in the figure, by producing the plan of a side or diagonal to the trace, finding the corresponding elevation of the produced side and hence the elevation of the required point in it. For example the elevation of the point D is found by producing B_1D_1 or *bd* to meet the horizontal trace LM in G. The elevation of G is g' on XY and Bg' is the elevation of the line BG.

Problem. XIX—Given the projections of any plane figure ABC and one projection $e'd'$ of a line ED in that plane, to determine the other projection. (Fig. 38.)

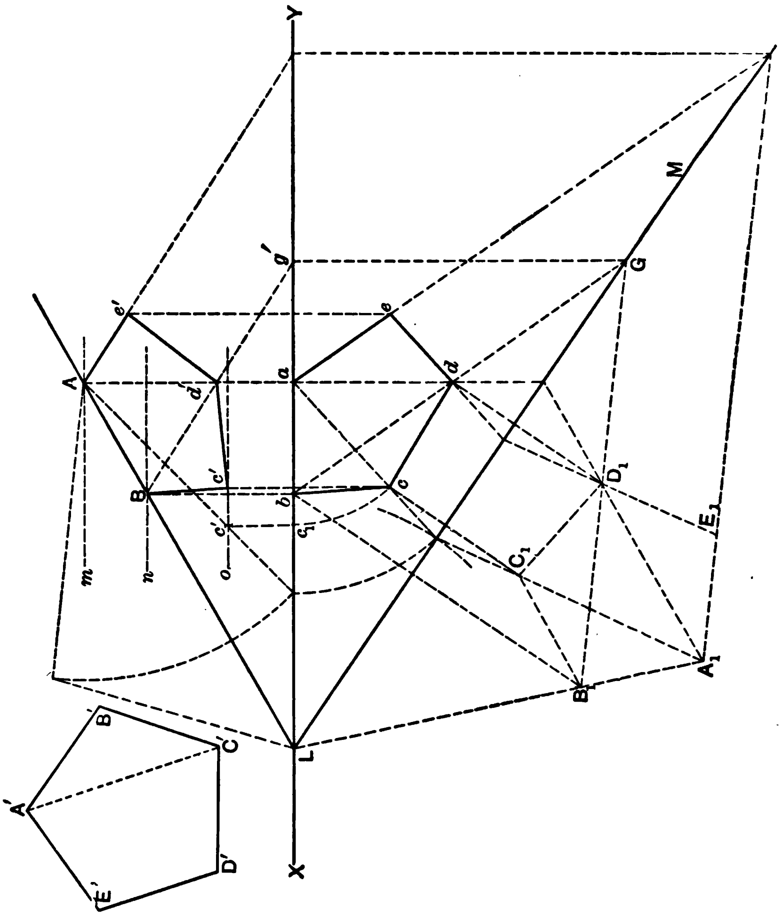


FIG. 37.

The vertical projection of the point of meeting of AB and DE is *m′*. The plan of this point must lie in *ab*, and is, therefore, at *m* where the perpendicular to XY through *m′* meets *ab*. Similarly, the point of meeting of BC and DE which has its elevation in *n′* must have its plan at *n* in *bc*. *m* and *n* are, therefore, two points on the plan of the line ED.

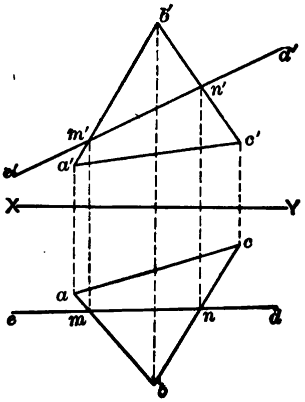


Fig. 38.

Where two lines meet, the line joining the intersection of the plans to the intersection of their elevations must be perpendicular to XY.

Problem XX.—To determine the projections of a circle when its position is given. (Fig. 39.)

Projections of circles are determined by fixing the projections of points in their circumferences and drawing freehand curves through the points thus projected. The projections of circles are, in general, ellipses.

Let it be required to find the projections of a circle when its plane is inclined to the horizontal at 60°, and is perpendicular to the vertical. Place the circle first in a horizontal position and divide the circumference equally in *aefd,* etc. The vertical projection is a line *a′b′* equal to the diameter; find in it the elevations of the several points. Now swing the elevation into the second position of the figure, inclined at 60° to XY. This represents the elevation in the required position, since the circle is still perpendicular to the vertical. By dropping perpendiculars from the several points in the vertical projection to meet lines parallel to XY, drawn through the corresponding points in the first plan, the plans of the several points in the new position are found. Draw a free-hand curve through the points thus obtained.

If it is required to find an elevation on a plane which is not perpendicular to the plane of the circle, a new XY may be

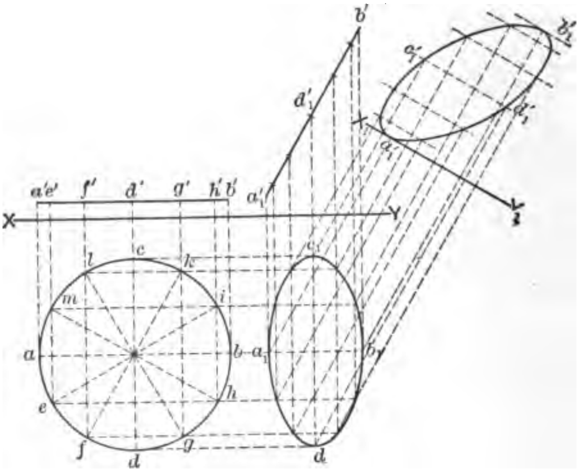


Fig 39

drawn, and the required elevation obtained by the method of Prob. III, Chap. I. (Fig. 8.) This drawing is obtained in Fig. 39, on a V.P. making an angle of 30° with its first position. The construction may be readily followed by reference to the method employed in Fig. 8 or Fig. 20.

Exercises on Chapter II.

1. Find the plan and elevation of a square of 1″ side, (*a*) when in the horizontal with a diagonal perpendicular to the V.P.; (*b*) when its plane is perpendicular to the V.P., but inclined to the H.P. at 60° and one angle is in the H.P.;

(*c*) when its plane is inclined at 60° to the H.P. and a diagonal which is horizontal makes 45° with the V.P.

2. Construct plans and elevations of a pentagon in each of the positions described in *a*, *b*, and *c*, Ex. 1.

3. Find the plan and elevation of a square of 1″.5 side when one of its diagonals is at 30° to the V.P. and 45° to the H.P., and the other diagonal is parallel to the H.P.

4. Find the traces of the plane containing the square in Exercise 3.

5. Find the plan of a circle of 1″.75 diameter when its surface is inclined to the horizontal at 45°, and find the vertical projection on a plane making an angle of 30° with the horizontal diameter of the circle.

6. Three points form, in plan, the angles of an equilateral triangle *abc* of 1″.5 side. The heights of the points A, B, and C above the H.P. are respectively 0″.5, 1″.0, and 1″.25. Find the vertical projection and the true form of the triangle (*a*) when BC makes an angle of 30° with XY; (*b*) when *bc* makes an angle of 45° with XY; (*c*) when *ab* is parallel to XY.

7. The plan of an ellipse is a circle of 2″ diameter on the H.P. One end of the major axis touches the H.P. and the other extremity of the major axis is 1″.0 above the H.P. Find the ellipse.

8. Find a plane parallel to the plane in Exercise 4 and at a distance of 1″ from it.

9. Find the projections of the four lines which pass through a point 1″ in front of the V.P. and 1″.5 above the H.P. and which are inclined at 40° to the H.P. and 30° to the V.P. (*a*) Find the traces of the four planes which are perpendicular to these lines and which pass through the given point. Draw each case as a separate problem.

10. Measure the angles which each of the four planes in

Ex. 9 makes with the projection planes. (They should be at 50° to the H.P. and 60° to the V.P.)

11. Measure the angles between each of the adjacent pairs of planes in Ex. 9.

12. Let a line be inclined at 30° to the H.P. and lie in a plane which is at 45° to the H.P. and perpendicular to the V.P. Find the two planes which contain the line and make angles of 60° with the plane.

13. Let the plane in Ex. 12 be inclined at 45° to the H.P. and 60° to the V.P., and contain the line at 30° to the H.P. Find the two planes containing the line and making angles of 60° with the given plane.

14. Find the angles which the line in Ex. 13 makes with a plane inclined at 45° to the H.P. and perpendicular to the V.P.

15. From one of the angles A of the triangle in Ex. 6 draw a line to meet BC at an angle of 45° in each of the cases *a*, *b*, *c*.

16. Draw the plan of an equilateral triangle of 2′ side when one side is parallel to the V.P. and at 30° to the H.P., and another side is at 45° to the H.P. (*a*) Find the elevation of the triangle on a plane making an angle of 30° with the side which is inclined to the H.P. at 30°.

17. Find the traces of the plane in Ex. 16*a* and measure the angle between the traces.

18. Draw the plan of a hexagon of 1″ side when three consecutive angles are at elevations of 2″, 1½″, and 1″, respectively, above the H.P. (*a*) Find the elevation on a plane containing one side of the figure.

CHAPTER III.

SOLIDS.

THE projection of solids may be regarded as the projection of the plane faces which form the outer surfaces of the solids. These plane faces are represented by lines, and the lines by the points which are their extremities. These extremities of lines are at the trihedral angles of the solids, so that the projection of solids is really only the projection of a series of points.

DEFINITIONS.—A solid has length, breadth, and thickness and is completely bounded by surfaces which may be plane or curved. Those which are bounded by plane faces are *polyhedrons*.

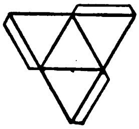
When the faces are all equal and the angles between any two adjacent faces are the same, the solid is a regular polyhedron.

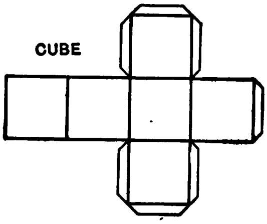
There are five regular polyhedrons, as follows:

The *Tetrahedron* has four equilateral triangular faces.

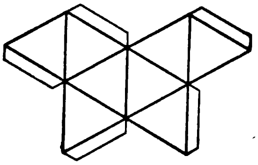
The *Cube* has six square faces.

TETRAHEDRON

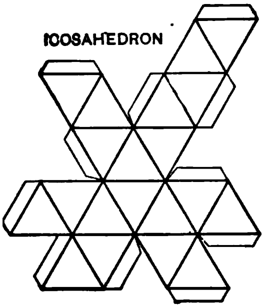




The *Octahedron* has eight equilateral triangular faces.

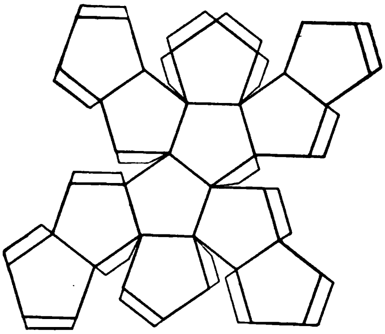


OCTAHEDRON



ICOSAHEDRON

The *Dodecahedron* has twelve regular pentagonal faces.



DODECAHEDRON

The *Icosahedron* has twenty equilateral triangular faces.

The surfaces of all these solids can be unfolded or developed into flat surfaces, and hence from flat surfaces models of them can be made. The developments are represented in the above diagrams. These developed surfaces should be accurately drawn on an enlarged scale, and the solids formed from them. Somewhat stiff drawing-paper should be used. In the developments, margins are left on some of the sides for fastening. After the pattern is cut out mucilage should be put on the margins and allowed to dry. To form the solids, bend the paper along the edges and damp the gum on the margins to fasten them. The paper will bend more readily if a sharp knife, guided by a straight-edge, has been run along the lines, cutting about half-way through the paper.

A *prism* has ends or bases which are parallel to each other, and has parallelograms for sides.

The *axis of a prism* is the straight line joining the centres of the ends.

A *right prism* has its ends perpendicular to its axis, and an *oblique prism* has its axis inclined at any angle to its ends.

The *altitude* of a prism is the length of the perpendicular line drawn between its ends.

A *pyramid* has a polygon for its base, and its sides are triangles having a common *vertex* or *apex.*

The *axis* of a pyramid is the line joining its apex to the centre of its base.

A *right pyramid* has its axis perpendicular to its base, and an *oblique pyramid* has its axis oblique to its base.

The *altitude* of a pyramid is the perpendicular distance between its apex and its base.

If a prism has the number of its sides increased to infinity, the outline of the base becomes a curve, and the solid is called a *cylinder.*

A *circular cylinder* is one of which any section at right angles to the axis is a circle, and a *right circular cylinder* has circular ends at right angles to its axis. A *right circular cylinder* may also be defined as a solid marked out by a rectangle revolving about one of its edges.

A *cone* bears the same relation to a pyramid that a cylinder does to a prism.

A *circular cone* is one of which any section at right angles to its axis is a circle, and a *right circular cone* has its base perpendicular to its axis. The latter may also be formed by a right-angled triangle revolving about one of the sides containing the right angle.

A *sphere* is described when a semicircle revolves about its diameter.

The *cylinder*, *cone*, and *sphere* belong to the class called *surfaces of revolution*, which are considered under *curved surfaces* in Chapter IV.

PROBLEM I.—To find the projections of a cube when one face is in the H.P. and the vertical faces make any given angles with the V.P. (Fig. 40.)

Let the angles which the vertical faces make with the V.P. be 30° and 60°. Since the four faces are vertical the plan will be a square. Draw the square $abcd$, ab and bc being at the required angles of 60° and 30° with the vertical. Let the side ABCD be in the horizontal. Its vertical projection is $a'b'c'd'$ in XY. From these points erect perpendiculars $a'e'$, $b'f'$, $c'h'$, $d'g'$, each equal to the length of a side of the cube, to meet the horizontal line $e'h'$ representing the upper surface of the cube EFGH.

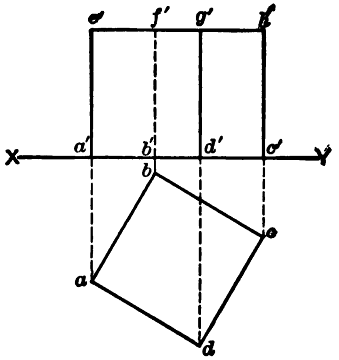


FIG. 40.

PROBLEM II.—To find the projections of a cube when one edge which is in the H.P. makes a given angle with the V.P., and two parallel faces make a given angle with the H.P. (Fig. 41.)

Let the edge in the horizontal make an angle of 30° with the V.P., and two faces make 60° with the H.P.

First place the cube with the faces inclined at the required angle to the horizontal, but perpendicular to the vertical. Draw the elevation $a'b'c'd'$, and from it obtain the plan *bfhd* in a manner precisely similar to the last problem; the elevation now taking the place of the plan, and vice versa.

Now take a new position for the V.P., i.e., change the position of XY so that the V.P. makes the required angle of 30° with AE or *ae*. From the several points in the plan erect perpendiculars to X_1Y_1 and on these perpendiculars measure, from the former elevation $a'b'c'd'$, the altitudes of the several angles above the horizontal. Thus to find the new elevation of the point D make the distance $m_1d'_1 = md'$. The point A being in the horizontal has its elevation at a'_1 in X_1Y_1; the elevation of B is found by making $b'_1o_1 = b'o$ and similarly for the point C. Joining these, the elevation of the face ABCD is obtained in $a'_1b'_1c'_1d'_1$ and the elevations of the other faces are obtained in a similar manner.

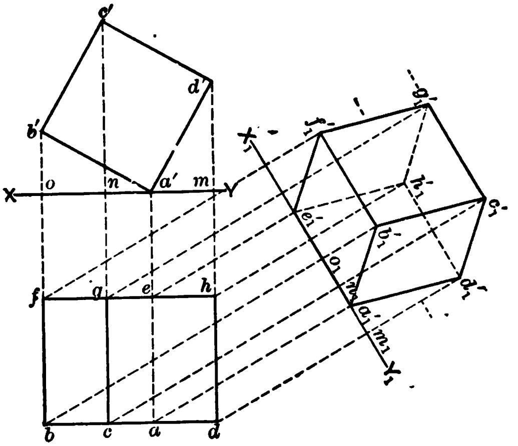


FIG. 41.

PROBLEM III.—To project a cube when one edge is inclined at any given angles to the projection planes and any given line in one face is horizontal. (Fig. 42.)

Let the given angles be 45° with the H.P., and 30° with the V.P., and let the diagonal BC be horizontal. First find the angles α and β which the projections of a line, equal in length to one of the sides of the square, at 45° to the H.P. and 30° to the V.P. make with XY by the method of Prob. III, Ex. 6, Chap. I. By the method of the previous problem, or more simply in this case, find directly the projections of the cube when one edge AE is inclined at 45° to the horizontal and the diagonal BC is perpendicular to the V.P. and, therefore, horizontal. Draw in the H.P. a square $A_1B_1C_1D_1$ representing one face of the cube, and having the diagonal B_1C_1 perpendicular to XY. Draw

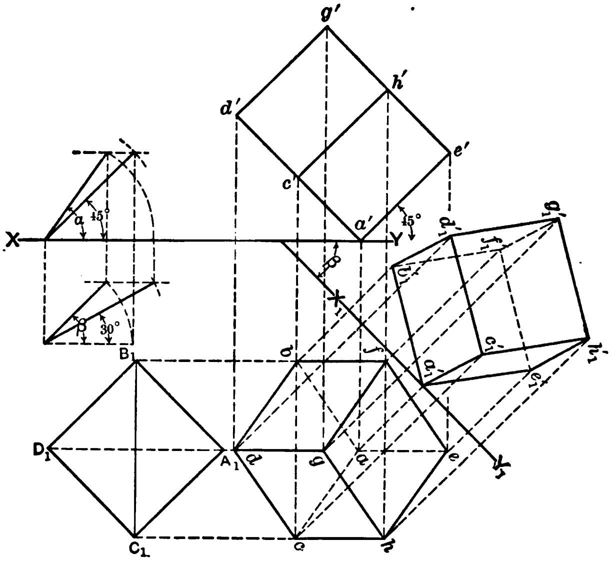


FIG. 42.

$a'd'$ equal to the diagonal of one face and bisect it in c', drop perpendicular lines from a', c', and d', and draw lines through A_1, C_1, and B_1 parallel to XY, meeting the former in a, b, c, and d. Join ab, bd, dc, ca to form the plan of the face ABCD, of which $a'c'd'$ is the elevation. Erect $a'e'$, $c'h'$, and $d'g'$ perpendicular to $a'd'$ (each equal to an edge of the cube), and join $e'g'$ to complete the elevation in this position, from which the remainder of the plan is readily completed. The length of ae, the plan of AE, will be found to equal the length of the plan of that line already obtained. Now take a new position of the V.P., so that the X_1Y_1 makes with ae the angle β, and from the plan and the altitudes of the several angles in the former elevation obtain a new elevation $a'_1b'_1c'_1$, etc., in the same manner as in the previous. problem. $a'_1e'_1$ will be found to make an angle with X_1Y_1 equal to α.

PROBLEM IV.—To project a cube when one of its faces is inclined to the H.P. at a given angle and an edge of that face is also inclined at a given angle. (Fig. 43.)

Let one face be inclined at 30° and an edge at 15°. By the method of Prob. XVI, Chap. II, obtain the projection of the face $abcd$ in the plane MLN inclined at 30° to the H.P. and having ad inclined at 15°. From the vertical projections $a'b'c'd'$ in ML erect the perpendiculars $a'a'_1$, $b'b'_1$, $c'c'_1$, $d'd'_1$, to the plane MLN and make them equal to the edges of the cube ab, bc, etc. These will be the vertical projections of edges of the cube, and $d'_1a'_1c'_1b'_1$ will represent the vertical projection of one face. From the vertical projection of the cube thus obtained and the plan of the face ABCD complete the plan of the cube. A projection on any other vertical plane may be obtained by the usual process of taking a new position for XY.

PROBLEM V.—To project a cube when one of its faces is inclined to the H.P. at a given angle and two of the angles of that face are at given altitudes with reference to the H.P.

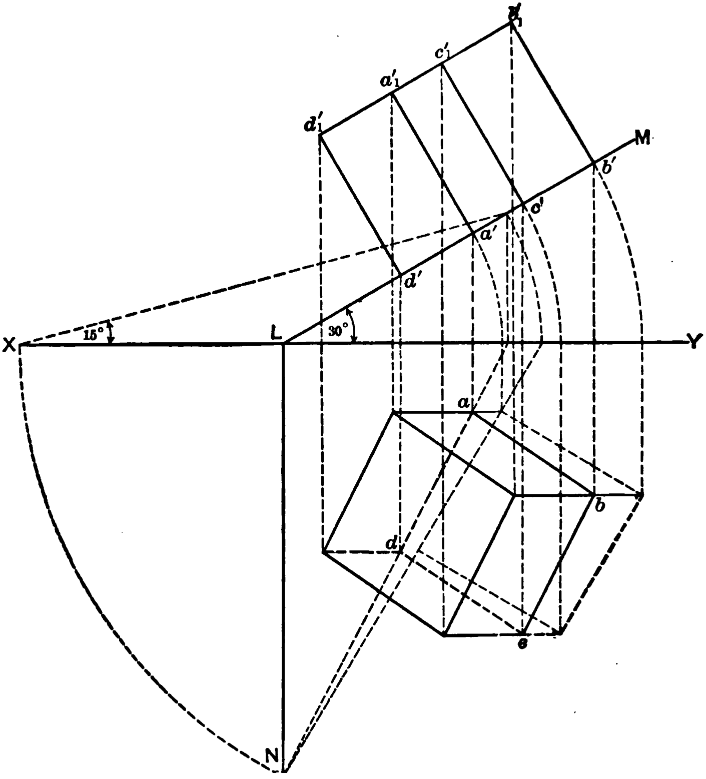


FIG. 43.

Project one face as in Prob. XVII, Chap. II, and from that complete the projection of the cube as in Prob. IV, Chap. III.

PROBLEM VI.—To project a cube when the altitudes of any three angles of one face with reference to the H.P. are given.

Project one face of the cube as in Prob. XVIII, Chap. II, and complete the projection as in the following problem.

PROBLEM VII.—To project a cube when the inclinations of two adjacent edges to the H.P. are given. (Fig. 44.)

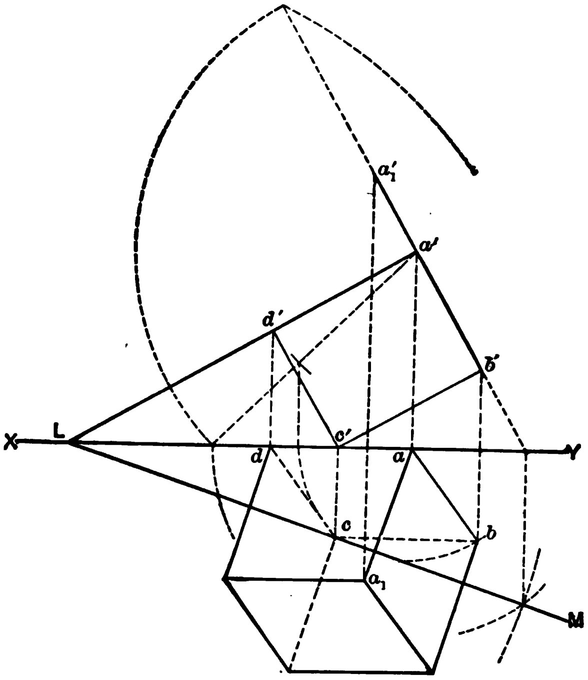


FIG. 44.

Let the two edges be inclined at 45° and 30° respectively. By the method of Prob. VIII, Chap. I, find the plan of one face in *abcd* and find its elevation in *a'b'c'd'*. The edges of the cube which are perpendicular to the face ABCD have their plans perpendicular to the trace LM and their elevations perpendicular to the trace L*a'*. Measure the projections of the length of one side in $a'a'_1$ and aa_1 (Prob. VI, Chap. I), and complete the projections of the cube. (The vertical projection is not completed in the figure.) In this case it is not necessary to rabat the plane in order to draw the plan. Where the figure is one with a large number of sides it may be necessary to do so as in Prob. XVIII, Chap. II.

Problem VIII.—To find the projections of a hexagonal prism when the axis is parallel to the vertical, but inclined to the horizontal at a given angle, and an edge of an end is in the horizontal. (Fig. 45.)

Let the axis be inclined to the horizontal at 45°. As a matter of convenience first draw the projections of the prism when resting on its base with two faces perpendicular to the vertical. Now draw the vertical projection with the axis inclined to XY, and from this projection and the plan of the end obtain the new plan *abcdjklg*. If it were required to find the projections with the axis at 45° to the V.P. and parallel to the H.P., what is represented here as plan would, in that case, be the elevation and *vice versa*.

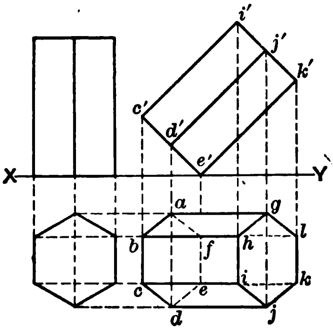


Fig. 45.

Problem IX.—To find the projections of a hexagonal prism when the axis is inclined to the horizontal at a given angle with one edge of an end in the horizontal and at a given angle with the V.P. (Fig. 46.)

Let the inclination to the H.P. be 30° and the inclination of the edge to the V.P. be 45°. First find the projections as in Prob. VIII, when the axis is inclined to the H.P. at 30°, but parallel to the V.P. Then draw X_1Y_1 at an angle of 45° with the edge *ef*, which is in the horizontal plane. From the several points in the plan erect perpendiculars to X_1Y_1 and obtain the altitudes of the elevations of these points by measurements as in Prob. II from the first found elevation. Thus the elevation of *a* is at a'_1 at a distance above X_1Y_1 equal to the distance of *a′* or *d′* above XY, and similarly for other points.

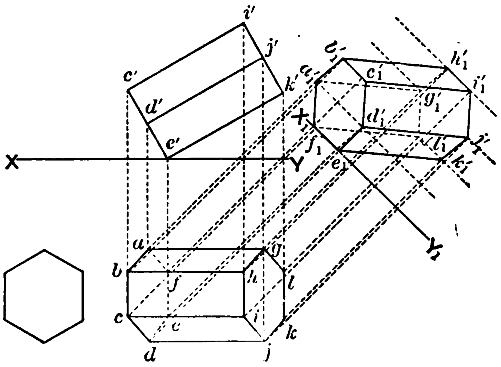


FIG. 46.

PROBLEM X.—To find the projections of a right pentagonal pyramid when one slant face is in the H.P. and the horizontal edge of the base is inclined at any angle to XY. (Fig. 47.)

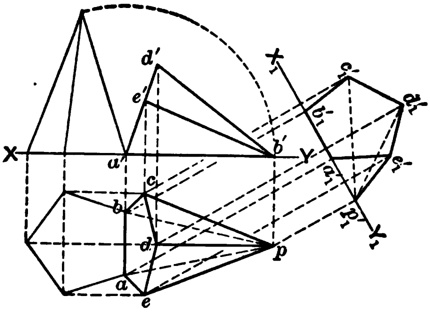


FIG. 47.

Let the altitude of the pyramid and length of the side of the base be given. First, project the pyramid standing on its base in the H.P. with one slant face perpendicular to the V.P. Then draw the elevation when the same face is in the H.P., and from this projection obtain the plan *abcdep*. Draw X_1Y_1 at the required angle, say 30°, to the side *ab*, and, as in former problems, obtain the new elevation in the required position.

Problem XI.—To project an oblique octagonal pyramid with its base in the H.P. (Fig. 48.)

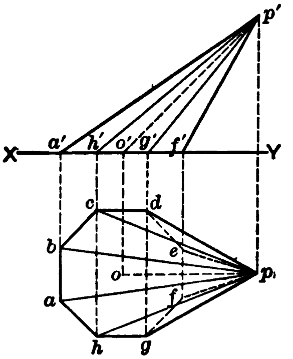


Fig. 48.

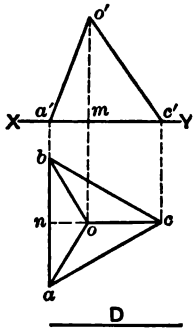


Fig. 49.

Let the base be a regular octagon, and let the axis, the length of which is given, be inclined to the base at 45°. Place the pyramid so that the plan of the axis is parallel to the vertical. Draw the regular octagon which forms the base. Find the elevation of the base and its centre. Through the elevation *o'* of the centre of the base draw a line *o'p'*, of the given length, at 45° to the horizontal. This will be the elevation of the axis. Join *p'* to the points *a'h'g'f'* to complete the elevation, and join *p* to *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *h* to complete the plan.

PROBLEM XII.—To determine the altitude of any regular pyramid when the lengths of the edges are known. (Fig. 49.)

Let it be required to find the altitude of a tetrahedron having a length of edge D. Draw the plan of the tetrahedron when one face is in the horizontal and an edge parallel to XY. In this case the plans of the slant edges bisect the angles of the base and BA is, therefore, perpendicular to the V.P. The vertical projection of OC is equal to itself. If, therefore, from *c′* an arc equal to *ab* be described to meet the perpendicular from *o*, the point of meeting *o′* is the vertical projection of the apex, and *o′m* is the altitude. The point *o′* may also be obtained by describing from *a′* an arc equal to *nc* to meet the vertical from *o* or the arc described from *c′*. Similar constructions may be made to obtain the altitudes of other pyramids, when the lengths of the edges are given.

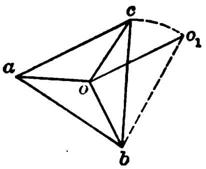


FIG. 50.

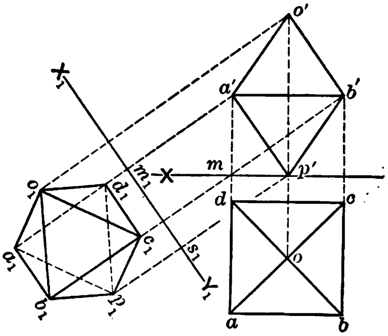


FIG. 51.

Another Method (Fig. 50) is to swing a vertical plane which contains one slant edge into the horizontal. In the tetrahedron, rabat the vertical plane *bo*O about *bo* into the horizontal; o_1, the rebatted position of O, is found by erecting a perpendicular to *bo* from *o* and describing an arc co_1 about *b* to meet the perpendicular in o_1. oo_1 is the required altitude.

PROBLEM XIII.—To draw the projections of a regular octahedron when one of the faces is parallel to the horizontal. (Fig. 51.)

The octahedron may be considered as being composed of two square pyramids base to base, having equilateral triangular faces. First place the solid so that the common square base *abcd* is horizontal and has two of its sides parallel to the vertical. In this position two of the three equal axes AC, BD are horizontal, and have their horizontal projections equal to their true lengths. The third axis OP is vertical, and since it is parallel to the V.P. its vertical projection $o'p'$ is equal to its true length. Now draw X_1Y_1 parallel to one of the faces ADP, i.e., to the vertical projection $a'p'$ of the face, and from this elevation and the plan in the first position obtain the new plan by dropping perpendiculars on X_1Y_1 and measuring distances from it equal to the similar distances in the old plan. Thus $m_1a_1 = ma$; $s_1p_1 = op'$, etc. The new-found plan shows two faces as equilateral triangles since these faces are parallel to the H.P.

PROBLEM XIV.—To project an octahedron when the inclinations of two of its solid diagonals are given. (Fig. 52.)

Let the edges of the octahedron measure 1″ and let the diagonals be inclined at 30° and 45°. Construct a square of 1″ sides and obtain the length of a solid diagonal. Mark off one-half this length in Ma_1, Mb_1 on lines inclined at 30° and 45° to the H.P. Project an angle of 90° as in Prob. VIII, Chap. I, in *amb*. Produce *am* and *bm* to complete the diagonals in *d* and *c*, and join the points thus found to obtain the projection of the "common base." Through *m* draw the third solid diagonal which is perpendicular to the plane *acdb* and its projections therefore perpendicular to the traces ML, LN. Find the projected lengths *mf*, *me*, and M*f'*, M*e'*, respectively, the plans and elevations of the third solid semi-diagonal lengths, and join the points *e*, *f* and *e'*, *f'* to *abcd* and *a'b'c'd'* to complete the plan and elevation of the solid. (For the sake of clearness the elevation has not been completed in the figure.)

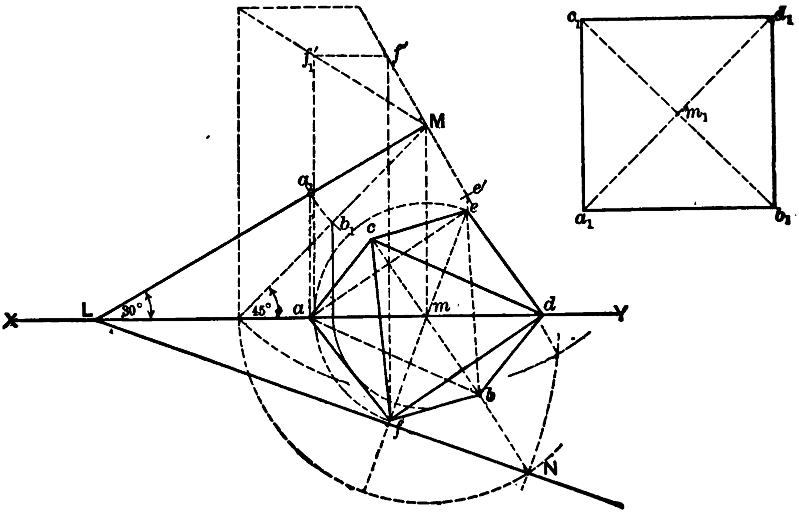


FIG. 52.

EXERCISES ON CHAPTER III.

Find the projections of the following:

1. A sphere 2″ in diameter when the centre is 1″.5 above the H.P. and 1″.75 in front of the V.P.

2. A sphere as above when its centre is 1″.5 below the H.P. and 1″.75 in front of the V.P.

3. A right prism whose ends are hexagons of 1″.25 side, and whose axis is 3″ long, lies with one side on the horizontal

plane. Draw its plan, and give an elevation on a plane which makes an angle of 30° with the axis of the prism.

4. Draw the plan of a cylinder (diam. 2″, axis 3″) when its axis is inclined at 45° to the H.P., and draw an elevation when the XY line makes an angle of 40° with the plan of the axis.

5. Draw the plan of an octahedron of 2″ edge resting with one face on the H.P., and an elevation on a plane not parallel to the plan of any edge.

6. A prism of 1″.5 square ends and 3″ long rests with an edge of an end in the horizontal plane; this edge makes an angle of 60° with the ground line, and the lowest long face of the prism makes an angle of 30° with the horizontal plane. Draw its plan and elevation.

7. An octagonal prism, 1″ side and 3″ long, has its ends inclined at 60° to the horizontal, and its long edges make angles of 15° with the vertical plane. Draw the plan and elevation when an edge of an end lies in the horizontal plane.

8. A pyramid has an equilateral triangular base of 2″ edge, and the other edges are 3″ long. Draw a plan and elevation when the axis makes angles of 30° with the vertical plane and 45° with the horizontal plane, one edge of the base being horizontal.

9. A hexagonal prism is 3″ long, and the greatest width of an end is 2″. Draw its plan and elevation when one of its long faces is at 15° to the horizontal plane, and its axis is at 30° to the vertical plane and horizontal.

10. A right cone has a base 2″ in diameter and a 4″ generating line. Draw its plan and elevation when the base is vertical and the axis is inclined at 45° to the vertical plane.

11. Project a right hexagonal pyramid when its base is

inclined to the H.P. at 45° and one of the edges of the base is inclined at 30°. The altitude of the pyramid is 3″ and an edge of the base 1″. (*a*) Find the vertical projection on a plane of which the XY makes an angle of 45° with the edge inclined at 30° to the H.P.

12. Project a regular tetrahedron of 1″.5 edge when two of the angles are at heights of 1″ and 1″.25 respectively above the H.P., and the plane of the base containing the angles is at 45° to the H.P.

13. Project, by the method of Prob. XVIII, Chap. II, a regular hexagonal prism 1″ side of base and altitude 2″, when three consecutive angles of an end are at heights of 1″, 1″.5, and 2″ above the H.P.

14. Project a regular octahedron of 1″ edge when one solid diagonal is at 45° and an adjacent edge is at 30° to the H.P.

CHAPTER IV.

CURVED SURFACES AND TANGENT PLANES.

Generation of Surfaces.—Geometrical surfaces are generated by a line, straight or curved, moving in a definite manner. This line is called the *generatrix* or *generating line.* A line which serves to define or direct the motion of the generatrix is called a *directrix.*

Surfaces of Revolution.—When a fixed curve revolves about a straight line as an axis so that all points in the curve describe circles, a surface of revolution is formed. Thus a semicircle revolving about its diameter describes a sphere. The sections of surfaces containing the axis are called *meridians* and represent the form of the generating line.

Ruled Surfaces are those that are generated by a straight line directed in any manner. They are divided into two classes, *developable* and *skew* or twisted surfaces.

Developable Surfaces are those which can be unfolded into a plane without tearing or crumpling. The cylinder and the cone are examples of this class.

The *hyperboloid of revolution,* which is generated by a straight line revolving about an axis not in the same plane with it, is an example of a *skew* surface.

A *screw* surface is formed by a straight line moving with a uniform velocity along a fixed straight line as axis and making a constant angle with the axis while it revolves uniformly about it.

A *cone* is a surface formed when one point in the generating line is fixed and that line is directed by any curved line.

A *right cone* is one of which any section perpendicular to the axis is a circle. This is also called a *cone of revolution.*

A *cylinder* is a surface generated by a straight line which remains parallel to the axis and is directed by any circle.

A *right cylinder* is one of which any section perpendicular to the axis is a circle. This is also called a *cylinder of revolution.*

A *tangent plane* at any point is one which touches all curved lines passing through the point and a *normal* to the surface at the point is a line perpendicular to this plane.

PROBLEM I.—To find the projections of a right cylinder.

(*a*) When the cylinder stands on the H.P. This is represented on the left of Fig. 53.

(*b*) When its axis is inclined at 45° and remains parallel to the vertical. This is represented in the central portion of Fig. 53.

It will be found convenient to divide the cylinder into twelve equal parts by lines parallel to the axis, as represented in the first position. These equally spaced lines may be copied on the second elevation, from the first, or they may be obtained directly by means of a semicircle described on one diameter. These lines being obtained, the cylinder is projected as if it were a prism having twelve faces, except that the projected points are joined by a free-hand curved line instead of straight lines. The plan in the second position is, therefore, to be derived from its elevation and the first-found plan, in the same manner as in former problems.

This plan may also be obtained by drawing a new XY at 45° to the axis of the first elevation, and from that elevation, and the distance of the first plan from the V.P., deriving the new plan by the method of Prob. XIII, Chap. III, and former problems.

(*c*) When the axis of the cylinder is at an angle to both planes. Let the inclination to the horizontal remain as in the second figure. Draw X_1Y_1 to make the required angle with the plan, so as to place the axis at the given angle to the vertical. (See Prob. III, Fig. 42.) From the plan in second part of the figure and altitudes obtained from its elevation, obtain the new elevation, and join the points

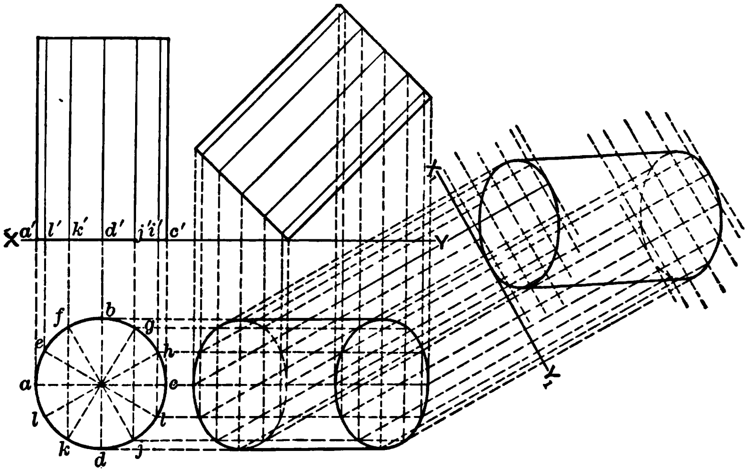


Fig. 53.

free-hand so as to obtain the required ellipses representing the ends of the cylinder.

The bounding lines representing the sides of the cylinder are drawn tangent to the ellipses. These lines are always at a perpendicular distance from each other equal to the diameter of the cylinder.

Problem II.—To project a right cone in any position.

The methods to be employed for the projection of cones are quite similar to those for cylinders. Tangent lines are to be drawn from the projections of the apex to the ellipses representing the bases. In Fig. 54 a cone is represented as

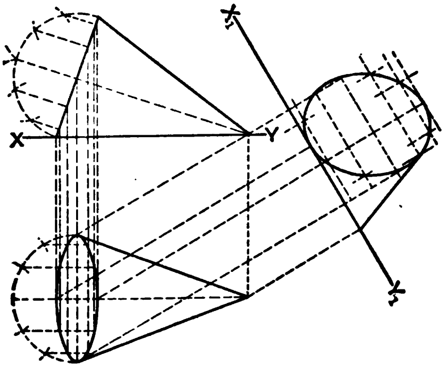


FIG. 54.

resting on the horizontal, with its axis parallel to the vertical, and also with its axis inclined at an angle to the vertical. If this angle be assigned, the angle which the axis in this position makes with the horizontal is to be measured, and the projections of a line making that angle with the H.P. and the required angle with the V.P. are to be obtained as in Prob. III, Fig. 42. The angle which the plan of the line so obtained makes with XY is the required angle between the plan of the axis and X_1Y_1.

PROBLEM III.—To draw a tangent plane to a cone. (Fig. 55.)

Let V be the apex of the cone; VR, VS two positions of its generator, and *rs* its horizontal trace. First let the point be on the surface of the cone and let a' be its elevation. Draw the elevation of the generator line through A to meet XY in b'. Find a point B in plan corresponding to the elevation b'; B*v* is the plan of the generator line passing through A, of which point a is the plan. The plane MLN containing vB and touching the trace in B will be the required tangent plane.

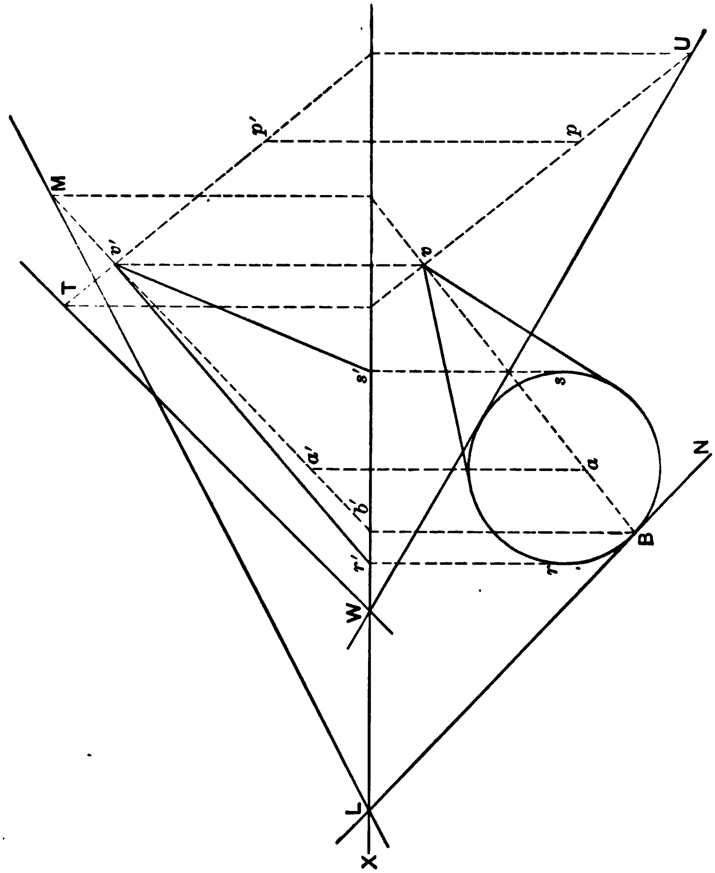


FIG. 55.

If the point through which the plane is to be drawn is external to the surface, as P, join P to the vertex V and find the traces of PV in U and T. Through U draw a tangent UW to the horizontal trace of the cone. This will be the horizontal trace of a tangent plane of which the vertical trace will be WT. A second tangent plane may of course be drawn through U touching the H.T. of the cone and containing T.

If it be required to draw a tangent plane to a cone parallel to a given straight line, the same construction as the foregoing may be employed when a line has been drawn through the vertex parallel to the given line.

PROBLEM IV.—To find a plane which shall contain a given line and make a given angle with a projection plane. (Fig. 56.)

Take any point V in the given line AB and drop a perpendicular VR on the H.P. Draw $v't'$ to meet XY at the angle α which the plane is to make with the H.P. Let VT be the generator line of a right cone and find its circular horizontal trace ST. The horizontal trace of the required plane must pass through A and touch the circle ST. One such plane is represented in ALB.

If it is required to find a plane which shall contain a given line and make a given angle with any plane, the same method is to be employed, but the problem becomes more difficult for the reason that the inclined planes on which the construction is to be made must first be rabatted into the projection planes.

PROBLEM V.—Given one projection of a generator of a right cylinder to find the other projection and the traces of the plane containing the line and being tangent to the cylinder. (Fig. 57.)

Let *abcd* be the plan and *efgh* the elevation of the cylinder, when it is parallel to both planes of projection, and let $m'n'$ be the elevation of a generator line. Draw LSL_1 to represent the trace of a plane perpendicular to both projection planes. Rabat this plane about its vertical trace into the vertical. The point of meeting of the axis of the cylinder and the cutting plane will be rabatted into q_1 and the trace of the cylinder into the circle $p't'$. Find the point p' in which $m'n'$ meets the rabatted trace of the cylinder and make qr equal to $p'r_1$. mn drawn through r parallel to ab or cd will be the plan of MN. Draw a tangent KL

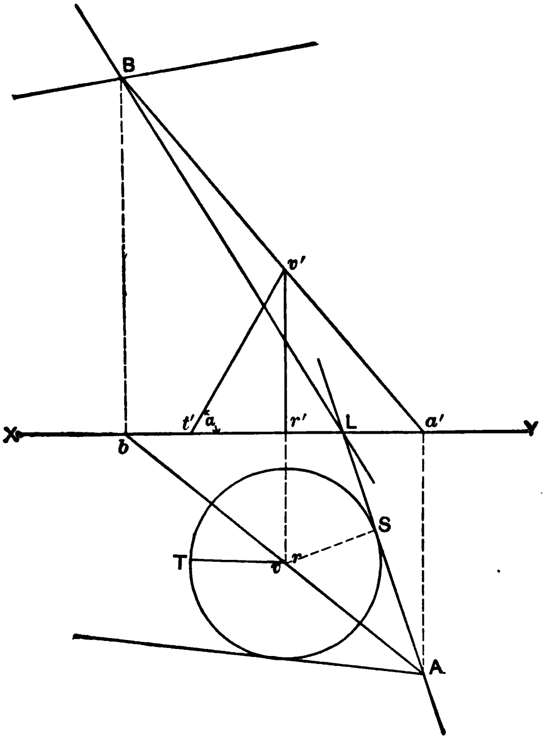


Fig. 56.

to the circle at p'. Make SL_1 equal to SK. LM and L_1N drawn parallel to XY will be the traces of the required plane.

When the axis of the cylinder is inclined to one of the planes a similar though slightly more involved method may be employed.

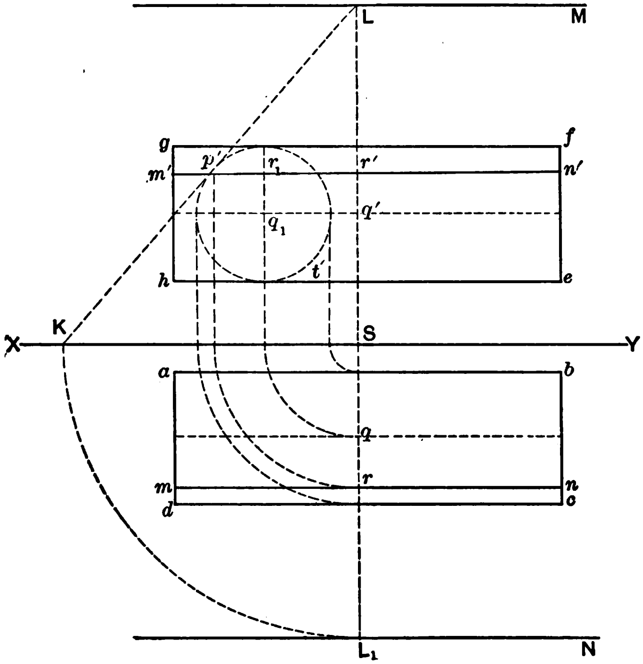


FIG. 57.

PROBLEM VI.—To project a helicoidal line, i.e., a line on a right circular cylinder which everywhere makes the same angle with the base of the cylinder. (Fig. 58.)

Such a line is formed on a cylinder by the inclined straight edge of a piece of paper, which is wrapped about the cylinder. When a straight line is at right angles to another straight line, and revolves about it uniformly, while at the same time it has a uniform motion in the direction of the axis of revolution, the revolving line describes a helicoidal or screw surface, and any point in the line describes a curve of the required form.

In Fig. 58 let *hg′* and *abc*, etc., be the elevation and plan of a right cylinder. Divide the plan into equal parts by radial lines, and find the corresponding vertical projections on the surface of the cylinder in $b'b'_1$, $c'c'_1$, etc. Let the pitch of the helicoid, i.e., the distance through which the generating line moves along the axis while it describes one complete revolution, be *hj*. Divide the length *hj* into the same number of equal parts as the circumference is divided in *a*, *b*, *c*, etc., in this case twelve. Through the points so found draw horizontal lines representing circles on the cylindrical surface.

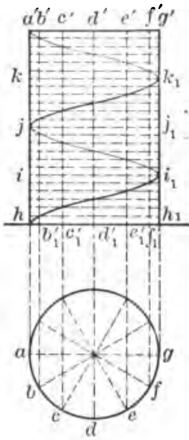
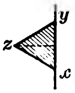


FIG. 58.



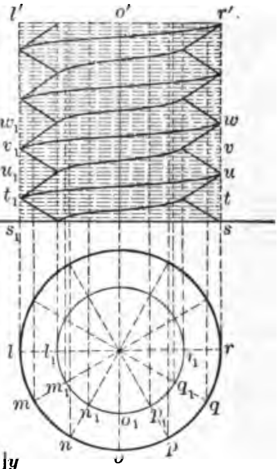


FIG. 59.

Suppose the describing-point to be at *a* in plan and *h* in elevation. When the point moves to *b* in plan, or, through

$\frac{1}{12}$ of a revolution, its elevation moves through $\frac{1}{12}$ of the pitch *hj*, i.e., to the horizontal line next above *h*. When *c* is the plan of the point the elevation is on the second horizontal line. When the plan is at *d* the elevation is on the third horizontal line, i.e., it has ascended through $\frac{3}{12}$ of the pitch, and so on. When one half a revolution is performed the elevation is at i_1. The vertical projection from *h* to i_1 is represented in a heavy line. For the remaining half-revolution when the point is behind the cylinder it is represented in a light line i_1j. A second revolution, which is similarly obtained, is represented in jk_1a'.

Problem VII.—To project a single screw-thread. (Fig. 59.)

Any line on the thread, such as its outer or inner edge, is a helicoidal line. Let the section of the thread be an equilateral triangle *xyz*. The pitch is equal to one side of the triangle. The plan *lr* represents the cylinders of contact at the tops and bottoms of the threads. The difference of the radii of the circles is equal to the perpendicular from *z* on *xy*. To trace the vertical projection of the screw, divide the circumference equally as in the previous problem. Mark off lengths *su*, *uw*, etc., equal to the pitch, and divide them into the same number of equal parts as the circumference, drawing horizontal lines through the points of division. Trace the vertical projections of the helicoidal lines on the outer and inner cylinder, as in the previous problem. Join the projections of the bottoms of the threads to the tops, forming equilateral triangles. For the sake of clearness in the drawing, only the front portion of the screw is shown.

Problem VIII.—To find the projections of a point on the surface of a sphere or other surface of revolution and to draw a plane through the point tangent to the surface. (Fig. 60.)

Assume one projection, as a'. A horizontal plane through a' will cut the sphere in a circle of radius $b'c'$. Draw the plan of this circle in a_1c. The plans corresponding to the elevation a' are a_1 and a_2. To find the tangent plane to the surface at A_1, draw a horizontal line through A_1 tangent to the small circle. The trace of this line M is a point in the trace of the required plane. Swing the vertical section containing A_1 about the vertical line passing through B until A_1 coincides with C, and draw a tangent to the curve at C to meet H.P. in N_1. Make bN equal to bN_1 or sn_1 and draw NL perpendicular to bN to form the required horizontal trace, and obtain the vertical trace passing through M by joining ML.

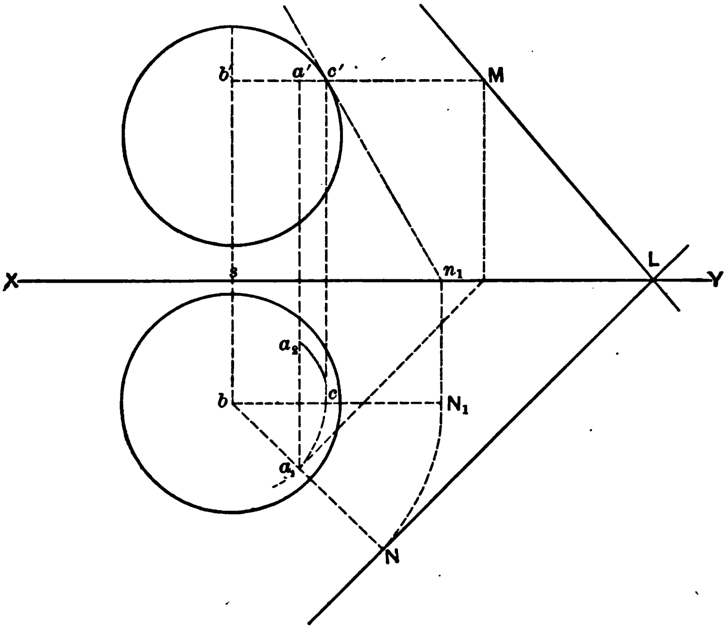


FIG. 60.

Problem IX.—Given the projections of four points, to find the sphere which shall contain the points.

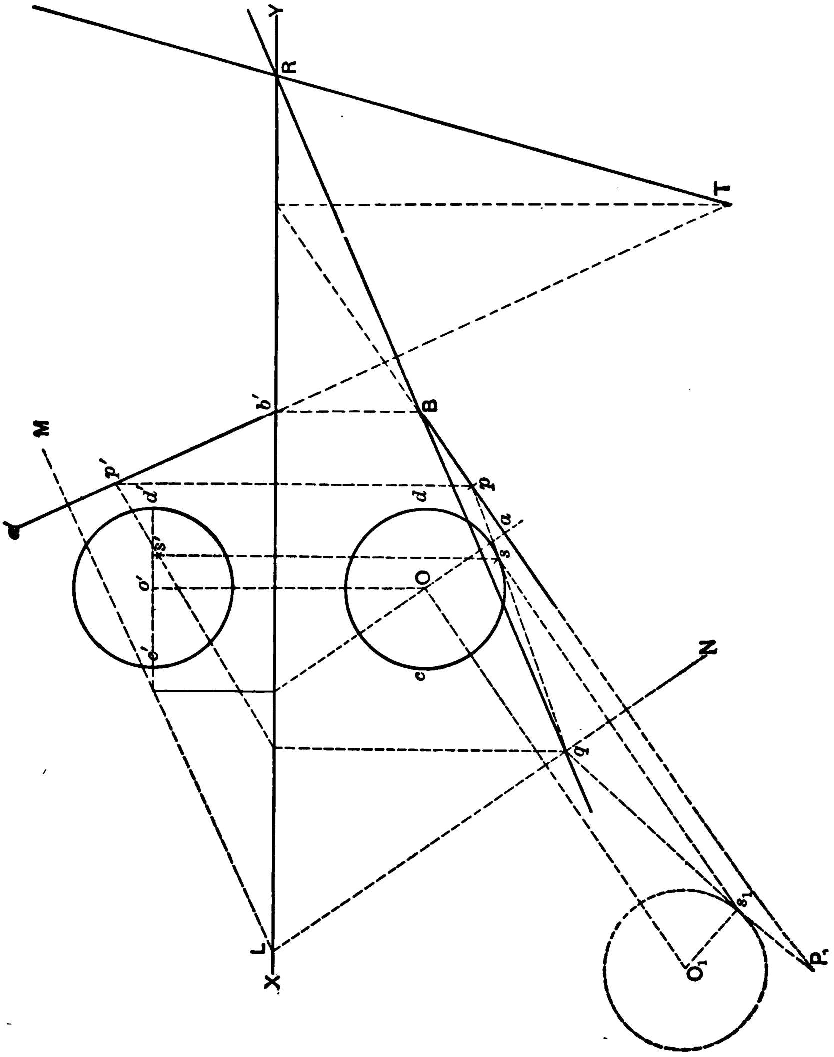
Let the four points be A, B, C, and D. Join any two of them, as AB, bisect AB, and through the point of bisection draw a plane perpendicular to AB. (Prob. VIII, Chap. II.) Bisect any other line, as BC, and through the middle point draw a plane perpendicular to the line. Find the line of intersection of the two planes thus found. This line must pass through the centre of the required sphere. Again bisect any other line, as CD, and find the plane passing through its middle point and perpendicular to it. Find the point in which the line of intersection of the first two planes meets the third plane. (Prob. VI, Chap. II.) This point is equidistant from all the given points and is the centre of the sphere. Find the true distance between the centre point and any one of the given points, and describe circles in plan and elevation to represent the required sphere.

Problem X.—To find the plane which shall contain a given line and be tangent to a given sphere. (Fig. 61.)

Let AB be the given line and CD the given sphere. Through O, the centre of the sphere, draw the plane MLN perpendicular to the given line AB. Find the point of meeting P between the line and the plane. Rabat the plane MLN about LN, and in the rabatted plane draw one of the tangents P_1q through P_1 to the great circle section. q will be a point in the horizontal trace of the required plane. As the plane must contain the line, B is also a point in the plane. The horizontal trace is therefore qBR and the vertical trace is RT. The plane touches the sphere in S.

Problem XI.—To find the plane which shall contain a given point and be tangent to two given spheres. (Fig. 62.)

Let P be the point and A and B the given spheres. The vertex of the cone which contains the spheres is found in


Y
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V by drawing tangents to the projections of the spheres. Any tangent plane to the spheres must contain V, and the required plane must also contain P. Join PV and by Prob. X (Fig. 61) find the plane NLM containing PV and touching one of the spheres A. Such a plane must also touch the other sphere B, since it must contain the vertex V of the cone which envelops it, and it is therefore the required plane.

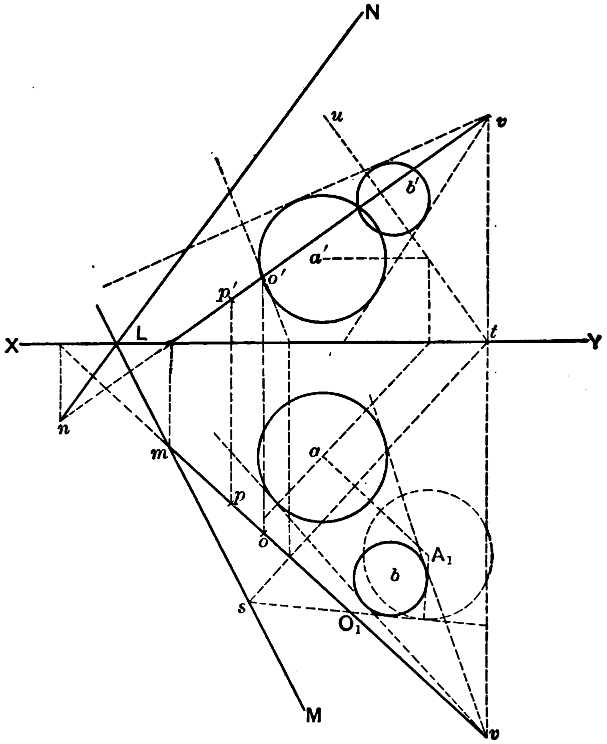


FIG. 62.

In Fig. 62 the construction required in order to determine the plane touching the sphere A and containing the line PV is given. *m* and *n* are the traces of the line PV, *stu* are the traces of the plane drawn through A perpendicular to PV, and O is the point of intersection between the line

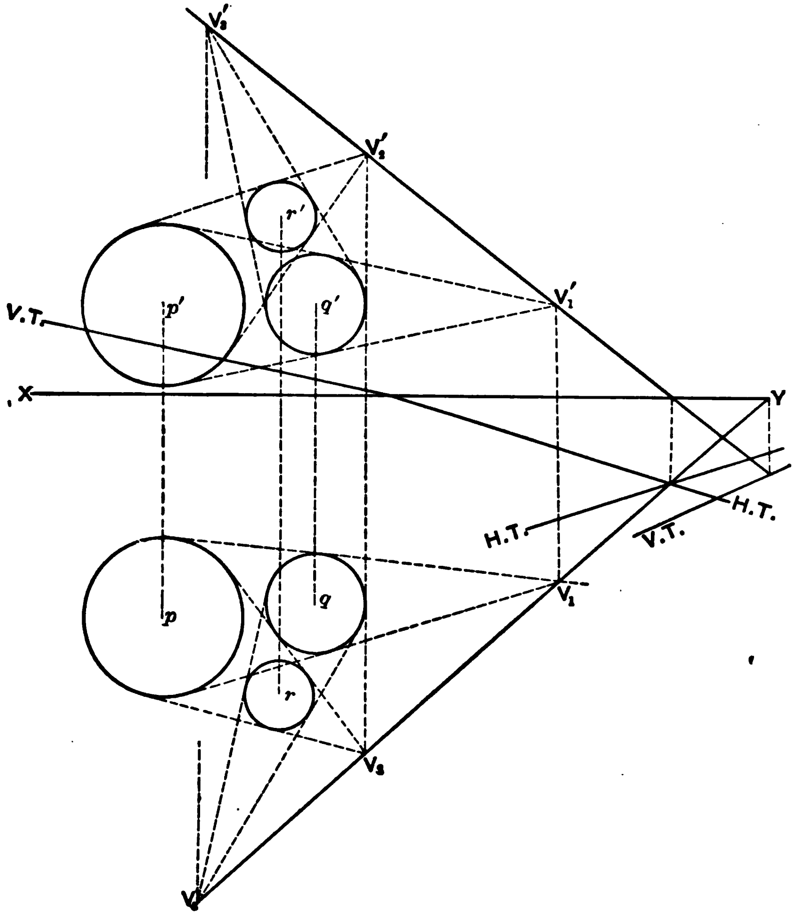


FIG. 63.

PV and the plane *stu*. A_1 is the centre of the rabatted circle A, and O_1 is the rabatted point O. The tangent line drawn from O_1 to the circle A_1 and produced to meet the trace determines the point *s* in the required tangent plane NLM.

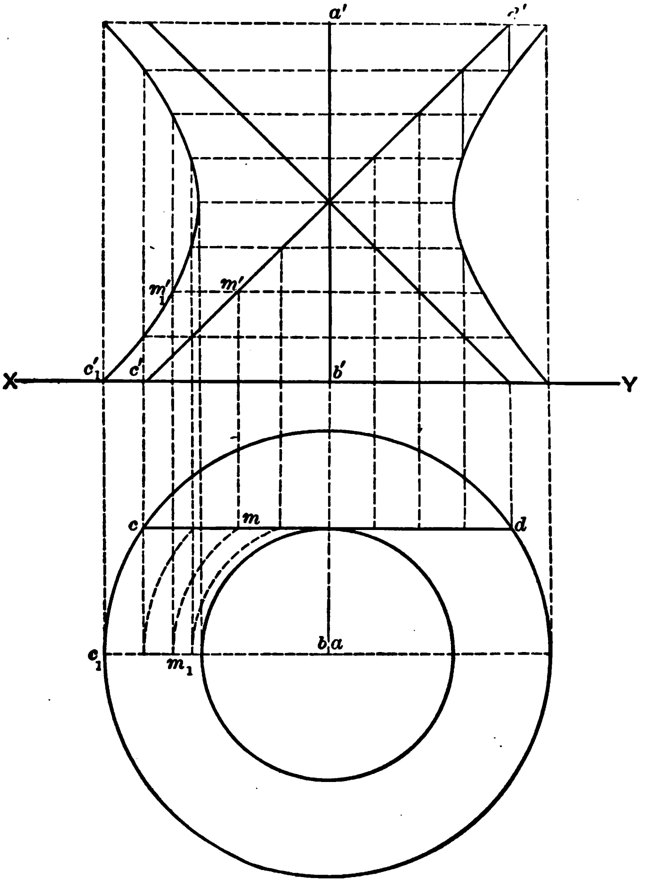


Fig. 64.

A second plane may also be found to contain the point and touch the spheres. As a cone having its vertex between the two spheres will also envelop the spheres, two other tangent planes (four in all) may be found to satisfy the problem.

Problem XII.—To find a plane tangent to three given spheres. (Fig. 63.)

Let the spheres be P, Q, and R. Call the vertex of a cone enveloping P and Q, V_1; the vertex of a cone enveloping P and R, V_2, and the vertex of the cone enveloping Q and R, V_3. These three points V_1, V_2, V_3 will be found to lie in a straight line. The plane containing this line and touching one of the spheres will touch all three spheres, and if the spheres are not in contact there will be eight tangent planes. Two of these containing V_1, V_2, V_3 are represented in Fig. 63.

Problem XIII.—To project an hyperboloid of revolution. (Fig. 64.) This surface is formed when a straight line revolves about an axis which it does not intersect.

Let the directing axis AB be vertical, and let CD be the generating line. Any point in CD will describe a horizontal circle about the directrix. Take any point M in CD and describe the arc mm_1 about a, and find the vertical projection m'_1 which will be a point on the curve representing the vertical section containing the axis. Similarly obtain other points and sketch in the curve.

Problem XIV.—To find the traces of a plane which shall be inclined at given angles to the projection planes. (Fig. 64*a*.)

Let the angle to the H.P. be 60° and that to the V.P. 50°. Let the point O in the XY line be the centre of any sphere *mn*. Construct a right cone touching the sphere and having its axis (SO) in the V.P., so that the elements of the cone are inclined to the H.P. at 60°. The horizontal trace of this cone will contain the arc h_1h_2. Construct also a right cone touching the sphere having its axis (TO) in the H.P. and elements inclined to the V.P. at 50°. The vertical trace of

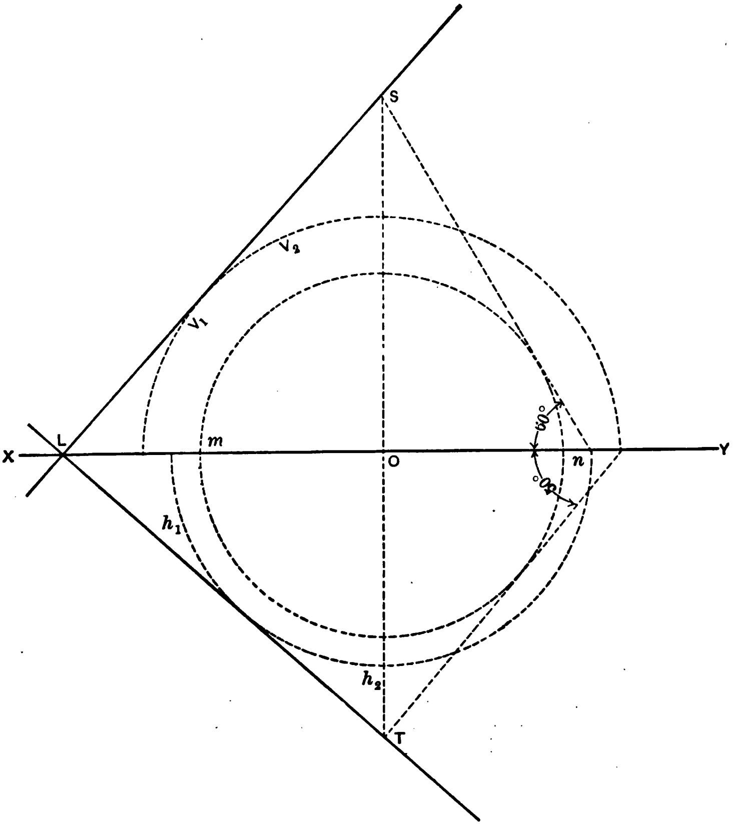


FIG. 64a.

this latter cone will contain the arc v_1v_2. The traces of a plane making the required angles will pass through S and T and be tangent to v_1v_2 and h_1h_2 as SLT. (See also Fig. 30.)

EXERCISES ON CHAPTER IV.

1. Find the projections of a line making angles of 30° with both planes of projection and the traces of a plane making an angle of 75° with the H.P. and containing the line.

2. A right cylinder has its axis parallel to the H.P. but inclined to the V.P. at 30°. The diameter of the cylinder is 1″ and its axis is 1″ above the H.P. Find the traces of the four planes which touch the cylinder and are inclined at 45° to the H.P.

3. The diameter of a screw-bolt is 3″; the thread is of square section, and the pitch ⅞″. Project the screw when its axis is parallel to the V.P.

4. A sphere of 2″ diameter rests on the H.P. and touches the V.P. Find the traces of a plane which will touch the sphere and make angles of 45° and 60° with the H.P. and V.P. respectively. (*a*) Find also the three other planes satisfying this condition.

5. Find the plan of a right hexagonal prism, faces 1″×3″, when the base is inclined at 45° and one face is at 60° to the H.P.

6. The base of a regular square pyramid is inclined at 60° and one of the triangular sides of the pyramid at 75° to the H.P. The edge of the base is 1″.5 and the altitude of the pyramid 3′. Find the plan; and find also an elevation on a plane perpendicular to the plane of the base.

7. Three spheres of 1″, 1½″, and 2″ diameter, respectively, are so placed that their centres are at the angles of an equilateral triangle of 2″ side and 1″ above the H.P.

The line joining the centres of the largest and smallest spheres is parallel to the V.P. and 2″ distant from it. Find the traces of all the planes which will touch all the spheres.

8. Determine the surfaces generated by the edges of a cube when it revolves about a solid diameter.

CHAPTER V.

SECTIONS OF SOLIDS.

When a plane cuts a solid, the resulting plane figure is called a *Section.* The section is usually given in its true form, but may be represented by its plan and elevation.

PROBLEM I.—To find any section of a right, square prism. (Fig. 65.)

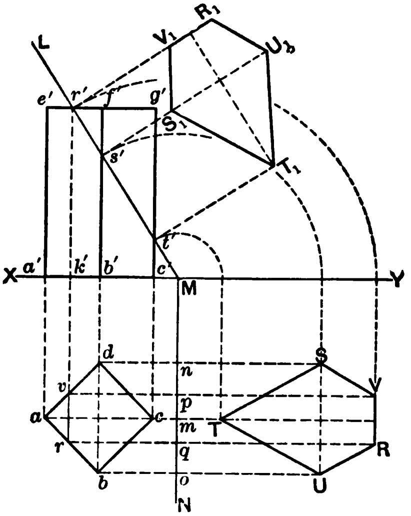


FIG. 65.

When the cutting plane is perpendicular to the axis the section is, of course, a square. When it is parallel to the axis the section is a rectangle. Let the section be required when the cutting plane meets the axis obliquely.

Place the prism so that the cutting plane shall be perpendicular to the vertical. Let the traces of the plane be LMN. The plane cuts the edges of the prism in points the projections of which are known. As that of which *c* is the plan and *t′* the elevation in the line CG; that of which *b* is the plan and *s′* the elevation in the line FB. These and the other corners of the section are readily represented by rabatting the cutting plane into the H.P. about MN. The point of which *c* is the plan and *t′* the elevation is rabatted into T on the perpendicular to MN drawn through *c*, and *m*T is equal to M*t′*. Similarly the point of which *b* is the plan and *s′* the elevation is rabatted into the point U, so that *o*U is equal to M*s′*. The point of which *d* is the plan is similarly rabatted into S, *n*S being also equal to M*s′*. The remaining pair of points are represented in elevation in *r′* and in plan by *r* and *v*, and their distances from the horizontal trace *p*V, *q*R are equal to M*r′*. The polygon TSVRU is, therefore, the true form of the section caused by the plane LMN. If the prism were longer, so that the edge AE met the plane, the section of the rectangular prism would be a lozenge-shaped figure.

The section may with equal facility be obtained by rabatting into the V.P. about the vertical trace LM. The perpendiculars $t'T_1 = c'c$; $s'S_1 = b'd$; $s'U_1 = b'b$; $r'V_1 = k'v$; and $r'R_1 = k'r$. The figures TSVRU and $T_1S_1V_1R_1U_1$ will be found to be precisely equal.

PROBLEM II.—To find the section caused by a plane meeting a pentagonal prism so that it cuts the axis obliquely. (Fig. 66.)

Let LMN, the cutting plane, be perpendicular to the V.P. The section may be obtained as in Prob. I by rabatting about one of the traces. Let it be turned about the horizontal trace MN. Then the plane meets the edge GD of the prism at a distance Mn' from the trace MN. Make $rN_1 = Mn'$ and N_1 will be the rabatment of the point of section. The other points are obtained similarly, and as in the previous problem, i.e., $sP_1 = Mp'$; $tQ_1 = Mq'$; $vO_1 = Mo'$; $uM_1 = Mm'$; and $M_1N_1P_1Q_1O_1$ is the required section.

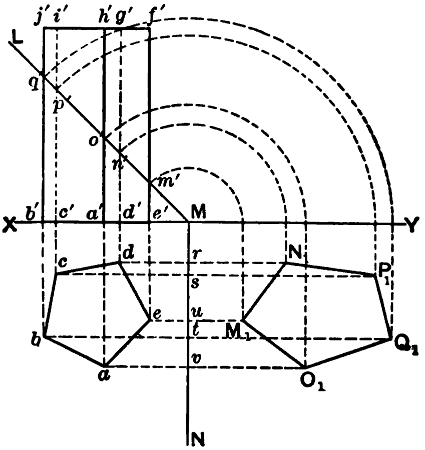


FIG. 66.

Where the given cutting plane is not perpendicular to one of the projection planes, the process of rabatting becomes more intricate, but the problem may be readily solved by first finding the points of intersection of the edges of the solid and the cutting plane by the method of Prob. 6, Chap. II (Fig. 26), and afterwards rabatting the section as in Prob. 23, Chap. II (Fig. 23). In such cases, however, it will sometimes be found simpler to obtain a new projection on a plane perpendicular to one of the traces and hence to obtain the rabatment as above.

PROBLEM III.—To find the section caused by a vertical plane meeting an hexagonal pyramid, when the latter rests on the H.P. on one of its slant faces. (Fig. 67.)

The horizontal trace cuts the plans of the edges of the pyramid in the points MNORPQ; find the elevations of these points in $m'n'o'r'p'q'$. The points are now easily rabatted into the horizontal since the altitude of each of them above the H.P. is known. $M_1N_1O_1R_1PQ$ is the true section.

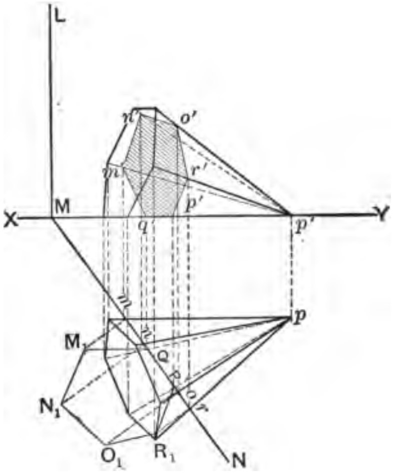


FIG. 67.

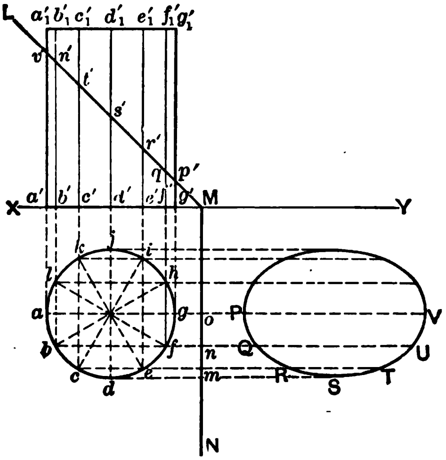
PROBLEM IV.—To find the section of a right cylinder by a plane which makes any angle with axis. (Fig. 68.)

Let the angle be 45°, and let the cylinder stand on the H.P. Take the cutting plane perpendicular to the vertical and inclined at 45° to the vertical axis of the cylinder. Divide the cylinder's circumference by the lines AA_1, BB_1, etc. (equally spaced for convenience). Rabat the plane into the horizontal about MN and obtain the section precisely as in the three former problems. $oP = Mp'$; $nQ = Mq'$; $mR = Mr'$, etc. Join the points free-hand.

Problem V.—To find the section of a cone which is caused by any plane.

(*a*) When the plane meets only one-half of the cone. (Fig. 69.)

Note.—The complete cone extends beyond the apex, so that what is below the apex O is only one-half of the cone. The cone extends indefinitely in both directions.


L
a'_1 b'_1 c'_1 d'_1 e'_1 f'_1 g'_1
v'
u'
t'
s'
r'
q'
p'
X
a' b' c' d' e' f' g'
M
Y
k
j
i
l
h
a
g
b
f
c
d
e
o
n
m
P
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R
S
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N

Fig. 68.

Divide the base of the cone into equal parts in *a*, *b*, *c*, *d*, etc. Join these points to *o*, the plan of the apex, and find the elevation of the lines in $a'o'$, $b'o'$, $c'o'$, etc. The plane will cross these lines in v', u', t', s', etc. Find the plans of these points in *v*, *u*, *t*, *s*, etc. In order to find the plan of V, in which case no intersection can be obtained, it is necessary to find the radius of the horizontal circular section at v', i.e., $v'm$, and describe a circle of that radius from *o*, or set off the distance $ov = v'm$. The other points *u*, *t*, etc., may also be obtained by describing circles from *o* with the radii of the horizontal sections at these points.

For example, the plan of U may be found by describing from the centre *o* an arc equal to *ln*, the radius of the horizontal section at U to meet the plan of the line OB in *u*. The plan and elevation of the section now being found in *vuts*, etc., *v'u't's'*, etc., it may be rebatted about either of the traces, as in the case of Fig. 65.

In this case, when the cutting plane meets only one-half the cone, the section is, in general, an ellipse. When the plane is perpendicular to the axis, the section is a circle, and when it is parallel to the tangent plane, i.e., when LM becomes parallel to a generator, as OD, the section is a parabola. The parabolic section is obtained in a similar manner to that of the ellipse.

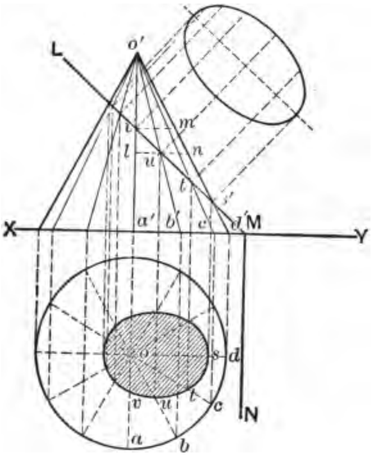

FIG. 69.

(*b*) When the plane meets both portions of the cone, the section becomes a hyperbola. This section may also be obtained by the same method as the ellipse. Let the section plane be rabatted about LM into the V.P. (Fig. 70). The line drawn through the point *v* parallel to the V.P. and being in the cutting plane is the axis of the hyperbola. Find the rabatted position of this line in w_1v_1. (The lengths $v'v_1$, $w'w_1 = Mv$.) The section may now be plotted by ordinates from this line. Let it be required to find the

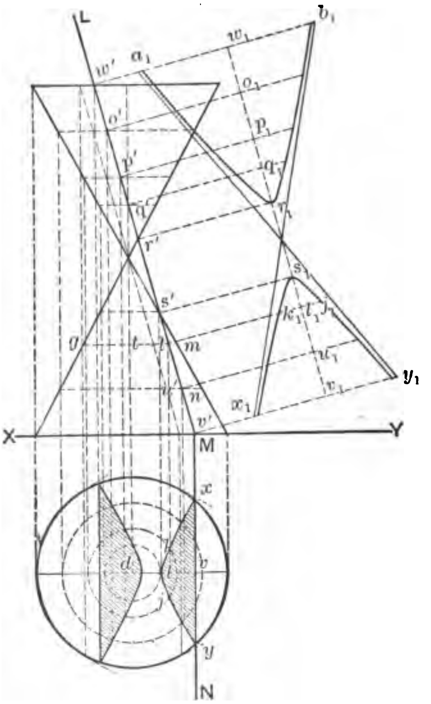

FIG. 70.

ordinates at l_1. Draw the horizontal line *gtlm* through the point *l* in elevation; and from the centre *d* in plan describe the circle with radius *tm* or *gt* meeting the perpendicular from *l* in *k* and *j*; *kj* is the double ordinate at the point l_1. Ordinates at the other points w', o', p', q', etc., are to be obtained in a similar manner.

To find the asymptotes to the hyperbola, draw through the apex of the curve a plane parallel to the plane LMN, and at the points in which the plane so found cuts the horizontal trace of the cone draw tangents to the circular trace to meet the trace MN in x and y. The line xy passing through v gives the length of the double ordinate to the asymptote at the point v_1. The ordinates w_1a_1, w_1b_1 at the point w_1 may be found in the same manner and the required asymptotes are the lines a_1y_1, b_1x_1.

Exercises on Chapter V.

1. Find the section of a triangular prism of which the right section is equilateral, caused by a plane perpendicular to one face and cutting that face at an angle of 30° to its longer edges.

2. A sphere of 2″ diameter is cut by a plane at a distance of 0″.5 from its centre. Find the section.

3. The altitude of an hexagonal right pyramid is 2″ and the length of one side of the base 0″.5. Find the section caused by a plane parallel to one edge of the base and to the axis and at 0″.25 from the latter.

4. The apex angle of a cone is 60°. Find the section caused by a plane which is inclined at 15° to the axis and passes 0″.25 from the apex.

5. Find the section of an octahedron of 1″ edge, caused by a plane meeting three adjacent edges at points distant respectively from a solid angle 0″.25, 0″.5, and 0″.75.

6. The axis of a right pentagonal prism measures 3″ and the edges of an end 1″. Find the section caused by a plane which enters the solid at one angle and which contains the opposite edge of the other end.

CHAPTER VI.

PENETRATIONS AND DEVELOPMENTS.

WHEN one surface cuts another, the form of the line of meeting or penetration is frequently required. This is usually first obtained by its projections and afterwards, when the surfaces are developable ones, represented in its true form. The form of the penetration lines between planes and solids in a few simple cases has already been dealt with under Sections of Solids in Chap. V.

PROBLEM I.—To develop the surface of an hexagonal prism which is cut by a plane meeting the axis at 45°, and find the line of penetration. (Fig. 71.)

Draw the plan and elevation when the axis is vertical, and suppose the solid to be cut by a plane LMN perpendicular to the V.P. First develop the whole surface of the prism by drawing the six rectangular faces, taking XY produced as a base, and measure on each of the vertical lines which represent the several edges of the prism the distances above the horizontal at which the cutting plane meets the edges of the prism. The vertical from d_1 is the developed position of DP, and the line of section is at a point l_1 such that $d_1l_1 = d'l$. The next edge, EQ, towards the left, represented by the vertical from e_1, is cut at a distance $e'g$ above the horizontal. Find the point of section in g_1, by drawing a horizontal through g to meet the vertical through e_1. Continuing towards the left, the point of section in

FR is found developed at i_1; in AM at k_1; in BN at j_1; in CO at h_1 and returning to the first-found point, which is represented again at l_1. The ends are not shown in the figure, but may be attached to any of the developed edges of the ends, as e_1f_1 or f_1a_1, etc.

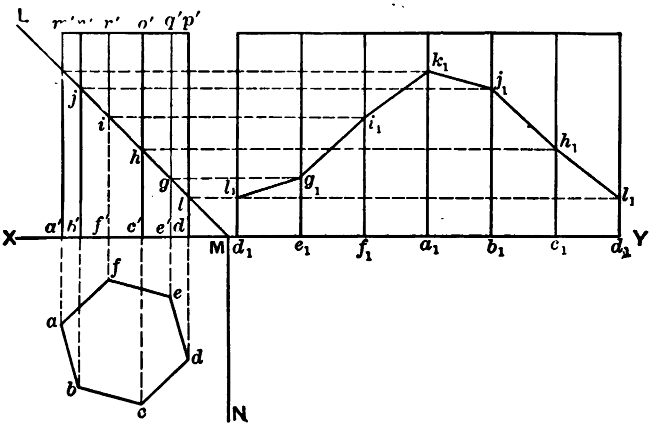

FIG. 71.

PROBLEM II.—To develop the surface of a right cylinder showing the penetration line caused by a plane meeting the axis of the cylinder at 45°. (Fig. 72.)

This problem may be solved by the same construction as in Prob. I.

Let the cylinder be placed upon its end and cut by the plane LMN as shown. Divide the surface of the cylinder by equally spaced vertical lines as shown in the figure. Measure off along XY produced, spaces equal to the length of the rectified arc of each division of the base, and, as in the former exercises, draw horizontal lines to obtain the points of penetration of each vertical, until the whole line of penetration $m_1n_1o_1p_1$, etc., is obtained.

PROBLEM III.—To develop the surface of a right cone showing the penetration line caused by a plane meeting the axis of the cone at 45°. (Fig. 73.)

Divide the base of the cone into equal parts in a, b, c, etc. From o' as centre, describe an arc of a circle with radius $o'a'$, the slant height of the cone. On this arc measure spaces $a'b_1$, b_1c_1, etc., equal to the lengths ab, bc, etc., and join o' to b_1, c_1, d_1, etc. The surface $o'a'a_1$ is the development of the cone surface, and the lines $o'a'$, $o'b_1$, $o'c_1$, etc., are the developments of the lines OA, OB, OC, etc. To find the developed line of section commencing at m, draw through n, t, p, q, etc., horizontal lines to meet the line $o'a'$. The true length of each of the projected lengths $o'n$, $o't$, $o'p$, etc., are thus marked off on $o'a'$. Now from o' as centre, describe with radius $o'k$ (equal to the true length ON) an arc to meet $o'b_1$ in n_1. Similarly describe the arcs lt_1, wp_1, etc., and join the points m_1, n_1, t_1, p_1, etc., to form the developed curve of penetration of the cone surface caused by the plane LMN.

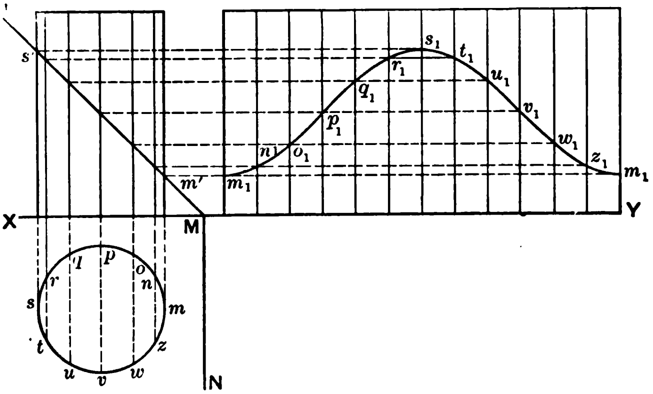

FIG. 72.

PROBLEM IV.—To find the projections of the lines of penetration of two square prisms and their developments. (Fig. 74.)

Let one prism be slightly larger than the other and let their axes meet at an angle of 60°. Place them so that the plane of the axes is parallel to the vertical and two edges of the smaller prism meet two edges of the larger. Draw the elevation and plan of the objects. The plans of the points of intersection will obviously be at *b*, *r*, *q*, *d*, *s*, and *t*. There are two points of section represented by each of the plans *b* and *d*; the elevations of the former are *v′* and *u′*, and of the latter *x′* and *w′*. The elevation corresponding to the plan *r* is *r′* since it must be on the line *i′l′*, which is the elevation of IL. Similarly the elevation corresponding to *q* is *q′*. *v′r′* and *r′u′* are the vertical projections of lines of section of which *br* is the plan and *x′q′*, *q′w′* are vertical projections

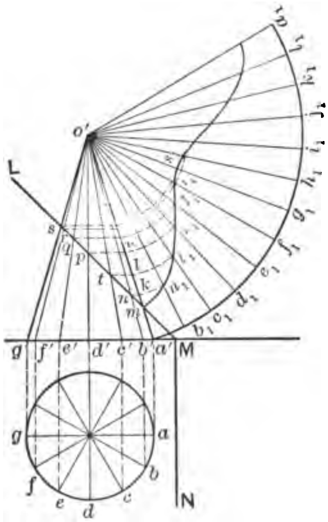

FIG. 73.

of lines of section of which *dq* is the plan. These lines may also be considered as the vertical projections of which *bt* and *sd* are respectively plans. If the position of the plan with reference to XY be changed, the projections of the penetration lines on opposite sides of the prisms will cease to overlap, and similarly, if the position of the vertical projection with reference to XY be changed, each penetration line will be separately represented in plan.

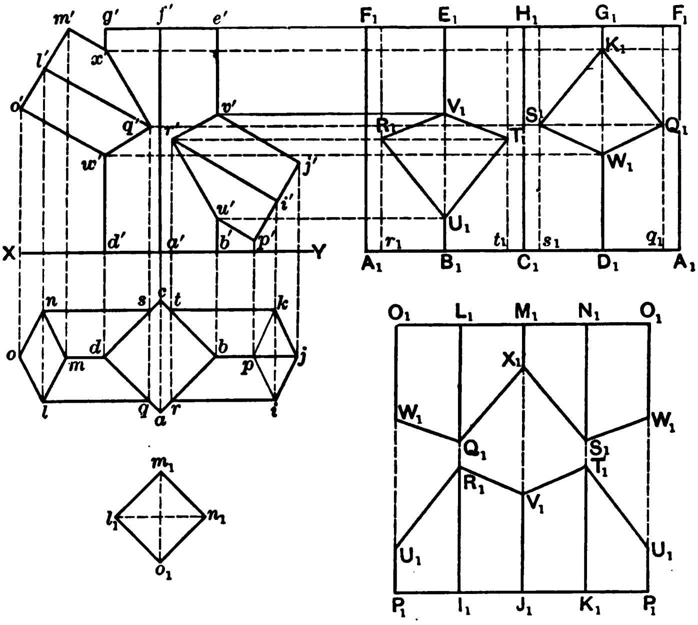

FIG. 74.

To develop the larger prism draw a straight line and mark off on it lengths A_1B_1, B_1C_1, C_1D_1, D_1A_1, each equal to the edges of the base. Erect perpendiculars at the several points equal to the length of the prism. The resulting rectangular figure represents the developed plane sides of the prism. Since the edges of the prisms are parallel to the V.P., all the projected lengths along these lines in the V.P. are true lengths. To find the point of penetration R, measure a distance A_1r_1 equal to *ar*, and erect a perpendicular r_1R_1, making its length equal to the altitude of r' in the elevation ($r_1R_1 = a'r'$). To find the points of penetration U and V, measure on B_1E_1 distances B_1U_1, B_1V_1 equal respectively to $b'u'$ and $b'v'$. To find the point of penetration T make B_1t_1 equal to *bt*, and erect a perpendicular at t_1, t_1T_1, equal to the altitude of T ($a'r'$). The points $R_1V_1T_1U_1$ being joined, show the lines of penetration. The penetration lines $S_1K_1Q_1W_1$ are obtained in a similar manner.

To obtain the penetration lines for the small prism, draw the four plane faces P_1L_1, I_1M_1, J_1N_1, K_1O_1, representing the developed sides. The projected lengths in the V.P., Fig 74, along the edges of this prism are true lengths since they are parallel to the plane. Make P_1U_1 equal to $p'u'$; I_1R_1 equal to $i'r'$; J_1V_1 equal to $j'v'$; K_1T_1 equal to $i'r'$, and again P_1U_1 equal to $p'u'$. The lines $U_1R_1V_1T_1U_1$ are the penetration lines of this end of the prism, and those of the other end $W_1Q_1X_1S_1W_1$ may be obtained in a similar manner.

PROBLEM V.—To draw the projections and developments of two right cylinders, the axes of which meet at any angles. (Fig. 75.)

Place the cylinders so that their axes are parallel to the V.P. Divide the surface of the smaller cylinder into equal parts by lines parallel to the axis through H, I, J, K, etc., and where these lines meet the larger cylinder in plan at *a*, *b*, *c*, *d*, etc., erect perpendiculars and draw the corresponding vertical projections. The horizontal projection of the line of penetration extends from *a* to *g* on the side of the large cylinder. The vertical projection corresponding to the point in the penetration, of which a is the plan, must be in the line $h'p'$ and is therefore at p'. A vertical projection corresponding to the point of which b is the plan, is in the line $i'q'$ and is therefore at q'. The other elevation corresponding to the plan b is in $z'o'$ and is therefore at o'. Similarly the projections of penetration points represented in plan by c and d are at r' and s' and at n' and m'. The curve $s'r'q'p'o'n'm'$ is the vertical projection of the portion of the penetration of which $abcd$ or $defg$ is the plan and therefore is the whole vertical projection of that end of the penetration in the position in which the cylinders are placed. The projection of the penetration at the other end is similarly obtained.

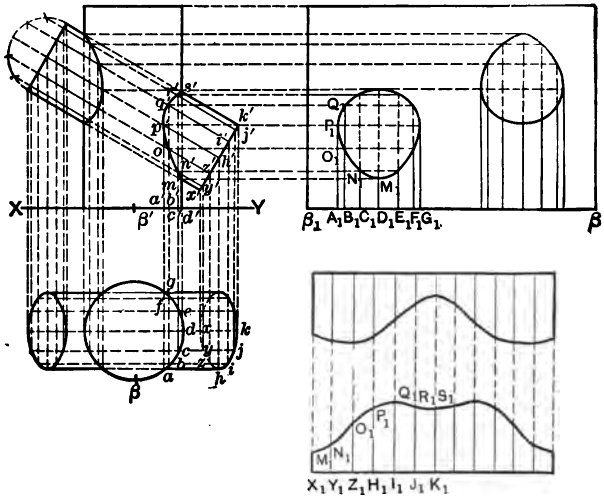

Fig. 75.

To find the development of the larger cylinder, use for convenience as a base line XY produced. Erect at β_1 a vertical line representing the vertical through β. Measure along XY, β_1A_1 equal to βa and lengths A_1B_1, B_1C_1, etc., equal to the lengths *ab, bc,* etc. The perpendiculars at the points A_1, B_1, C_1, etc., represent the several positions of the generator line at A, B, C, etc., and the whole rectangle is the developed surface of the sides of the cylinder. On the vertical through A_1 measure a height A_1P_1 equal to $a'p'$, which is the altitude of the point P above the H.P. On the vertical through B_1 measure the altitudes B_1O_1 and B_1Q_1 equal to $b'o'$, $b'q'$. These may conveniently be obtained by drawing horizontal lines through o' and q' to meet the vertical lines. The remaining points in this curve and the curve of penetration on the other side of the cylinder may be obtained in a similar manner.

To obtain the development of the curves on the smaller cylinder, draw a base line equal to the length of the circumference of the cylinder, marking the points X_1, Y_1, Z_1, H_1, etc., and erecting perpendiculars representing the several positions of the generator line at X, Y, Z, H, etc. Make X_1M_1 equal to $m'x'$; Y_1N_1 equal to $y'n'$; Z_1O_1 equal to $z'o'$, and find the other points P_1, Q_1, etc., to complete the curve. Draw the curve of penetration for the other end in a similar manner.

PROBLEM VI.—To draw the projections and developments of the curves of penetration between a cylinder and cone, the axes of which are respectively parallel and at right angles to the horizontal, but do not necessarily meet.

Let the cone and cylinder be placed as in Fig. 76.

The method which is used to find the lines of penetration consists in passing horizontal planes so as to cut both solids in lines of section which give intersections in points common to both solids. Horizontal planes are chosen because the resulting sections are simple, being circles for the cone and rectangles for the cylinder. First draw a horizontal plane

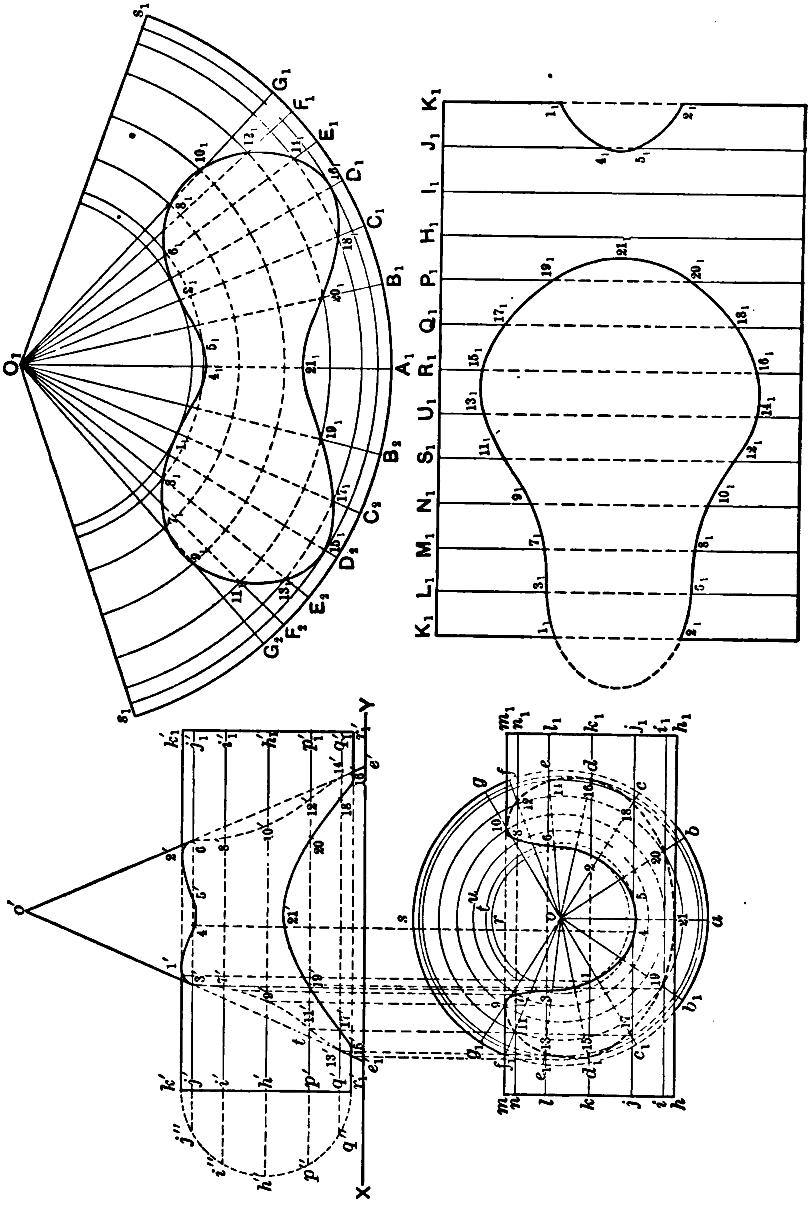
O$_1$
s$_1$
A$_1$
B$_1$
B$_2$
C$_1$
C$_2$
D$_1$
D$_2$
E$_1$
E$_2$
F$_1$
F$_2$
G$_1$
G$_2$
K$_1$
J$_1$
I$_1$
H$_1$
P$_1$
Q$_1$
R$_1$
U$_1$
S$_1$
N$_1$
M$_1$
L$_1$
K$_1$
o'
X
Y
k'
j'
i'
h'
p'
q'
r'
e'
j''
i''
h''
p''
q''
s
t
a
b
b$_1$
c
c$_1$
d
d$_1$
e
e$_1$
f
f$_1$
g
g$_1$
h
h$_1$
i
i$_1$
j
j$_1$
k
k$_1$
l
l$_1$
m
m$_1$
n
n$_1$
o
r
t
u

FIG. 76.

tangent to the cylinder along KK_1. This cuts the cone in a circle of which the horizontal projection is $r12$, and as the horizontal projection of the tangent line is kk_1, 1 and 2 are plans of penetration points of which the vertical projections are found at 1′ and 2′ on $k'k'_1$. Next let the solids be cut by a horizontal plane (containing JJ_1) which cuts the cylinder in the rectangle of which the plan is jj_1l_1l and the cone in the circle of which the plan is $t35$. The rectangle and circle meet in the points 3, 4, 5, and 6, which are therefore points in the plan of the penetration. The elevations of these points are 3′, 4′, 5′, and 6′ in $j'j'_1$. Similarly a plane through II cutting the cylinder in the rectangle II_1NN_1 and the cone in the circle $u78$ gives the points 7 and 8. Two points only are obtained in this case, as the circle only meets the rectangle in one line NN_1. In other words, the cylinder cuts the cone entirely away on one side at the level of II_1. The plane containing MM_1HH_1 serves to obtain the points 9 and 10. The plane containing PP_1 serves to obtain the four points 11, 12, 19, and 20, as here again the rectangle meets the circle in four points, i.e., the cylinder is again enclosed in the cone. The remaining planes being drawn and all the necessary points obtained, a free-hand drawing is made joining the points in plan and elevation.

To develop the cone surface: Through *o*, the plan of the apex, and as many of the points as are necessary to determine the form of the curve, draw lines *og*, *of*, *oe*, *od*, etc., to the base. These lines may also be drawn in elevation, although this is not necessary. Draw the line O_1A_1 to represent the developed position of the generator line OA, the true length of which is equal to $o'e'$. From O_1 as centre with radius O_1A_1 describe an arc of a circle, and on it measure from A_1 both ways lengths A_1B_1, A_1B_2 equal to *ab* or ab_1. From B_1 and B_2 measure B_1C_1 and B_2C_2 equal to *bc* and b_1c_1, etc., etc., until the several positions of the generator shown in plan are represented on the development. The developed position of the several points in the curve may now be obtained by describing arcs of the proper radii from O_1, representing the distances of the points from O, the apex of the cone, to cut the radiating lines. Suppose the point 19 is to be obtained; it will be found in the line O_1B_2 and at a distance from O_1 equal to the length of $O_1 19$, of which the plan is *o*19 and elevation *o*′19′. The true length *o′t* of O19 is found by drawing a horizontal through 19′ to meet $o'e'_1$ in *t*. With O_1 as centre and *o′t* as radius describe an arc to meet O_1B_2 in 19_1. 20 is at the same distance from O as 19, hence the developed position of 20 is found in O_1B_1 by continuing the arc passing through 19. 11 and 12 are also on this arc, since all these points 11, 12, 19, and 20 are in the same horizontal plane, i.e., at the same distance from the apex O. Similarly, all other points in the development may be obtained. Generator lines have not been drawn for all the points near the apex in order to avoid confusion. Their positions are, however, represented on the curve.

The development of the cylinder is obtained after the same manner as that in Prob. V. The straight line K_1K_1 represents the rectified circumference of the cylinder. The perpendiculars through K_1, L_1, M_1, N_1, etc., represent the equally spaced lines on the circumference parallel to the axis of the cylinder. Measure on the perpendicular through K_1 a distance $K_1 1_1$ equal to *k*′1′, which is a true length since it is a projection of a line parallel to the V.P. (The plan, *k*1, is also a true length.) Measure also the distance $K_1 2_1$ equal to *k*′2′. Similarly measure $L_1 3_1$ and $L_1 6_1$ equal to *j*′3′ and *j*′6′; $M_1 7_1$ and $M_1 8_1$ equal to *i*′7′ and *i*′8′, etc. The free-hand curve drawn through all the points thus obtained is the developed curve of penetration on the surface of the cylinder.

Problem VII.—To obtain the projections of the penetration lines between any two surfaces of revolution whose axes meet. (Fig. 77.)

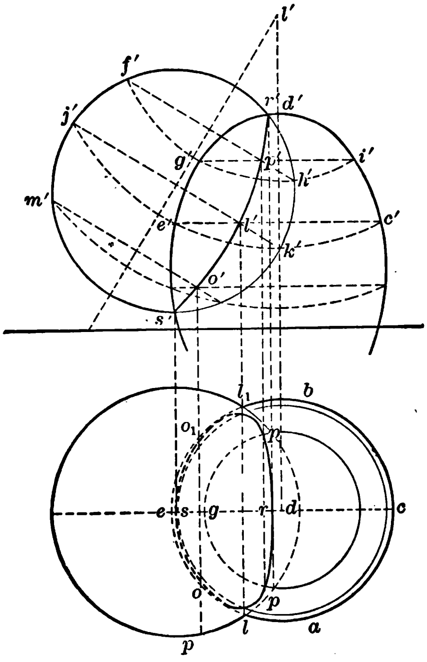

Fig. 77.

Let an ellipsoid whose axis is vertical be penetrated by a sphere.

An axis of the sphere may be taken in any direction. Take a diameter of the sphere meeting the axis of the ellipsoid in L. Let the vertical plane be parallel to the two axes. Let L be taken as the centre of a sphere, the surface of which meets the two solids; the section lines between this auxiliary sphere and the two solids are both circles, and having their planes perpendicular to the vertical, are projected there as straight lines. Let $f'g'h'i'$ represent the vertical projection of a sphere with centre L; $f'h'$ is the vertical projection of the circle in which this sphere meets the given sphere, and $g'i'$ the vertical projection of the circle in which it meets the given ellipsoid. These two circles meet in p', which is the projection of two points common to the two planes. In the same manner a sphere from the same centre, meeting the solids in JK and EC, gives the projection l' of two other points on the penetration line. The curve $r'p'l'o's'$ is therefore the vertical projection of the penetration line.

To obtain the plan: $g'i'$ is the elevation of a circle which is parallel to the horizontal, and its plan is therefore a circle. Describe this plan about the centre d, which is the plan of the axis of thc ellipsoid. Drop a perpendicular to find the plans p_1 and p corresponding to the elevation p'. Obtain in the same way the plans of the other points and join them. $rplos o_1 l_1 p_1$ is the projection of the penetration curve.

This method can be applied to a large number of cases, and is specially applicable to penetrations of spheres with other surfaces of revolution, since the axis of the sphere may be taken as any diameter.

PROBLEM VIII.—To obtain the line of penetration of any two conical surfaces. (Fig. 78.)

Let the two cones be V*mn* and W*qs*, having their horizontal traces given. These traces are conveniently taken as circles. Produce the line VW and find its horizontal trace in H. Any plane which contains VW will, if it cuts the cones, meet them along generator lines, and these lines will usually determine four points common to both surfaces and therefore on the line of penetration. Represent

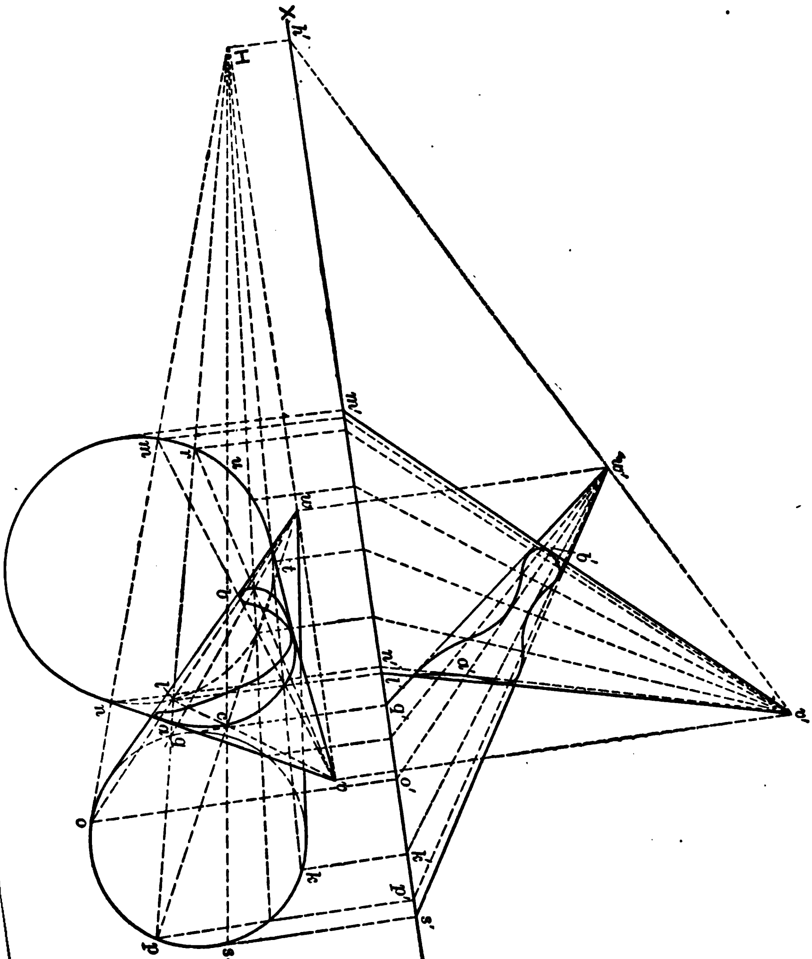

FIG. 78.

any cutting plane containing VW by its trace H*o*. This plane will cut one cone in the lines MV, NV, and will touch the other along WO. The meeting of these lines in plan will determine the points *a* and *b*, and in elevation *a′* and *b′*. Any number of planes may be similarly drawn, as, for example, H*p*, which will meet the cones in VR, VL and WQ, WP, and hence determine four new points.

It will be well, in order to avoid confusion, to proceed from one plane to another near to it, and to join the points as found, confining attention in the first instance to one projection only.

Where the trace lines of the two cones are given on different planes it will be necessary to obtain both the vertical and horizontal traces of the line VW and to employ both the traces of the cutting planes. The method will, however, be similar to the foregoing.

To develop the surfaces it is only necessary to measure the true lengths of lines from the apex to points on the penetration, and the true distances along this line between the adjacent points.

COR. I.—Where a cone intersects a cylinder the line (VW) common to planes which will contain generators of both surfaces will be drawn through the apex of the cone parallel to a generator of the cylinder.

COR. II.—Where two cylinders meet, the construction planes are drawn parallel to the generators of both cylinders.

EXERCISES ON CHAPTER VI.

1. A hexagonal prism is penetrated by a cylinder so that their axes meet at right angles. Let the axis of the prism be horizontal and two faces vertical. The diameter of the end of the cylinder is 1″.5 and the greatest diameter of the end of the prism also 1″.5. Find the lines of pene-

tration and develop both surfaces. (*a*) Show also a vertical projection when the axis of the prism is at 30° to the V.P.

2. A cube of 2″ side penetrates a sphere of 1″.5 radius so that their centres are coincident. Show the plan of the lines of penetration when a solid diameter of the cube is vertical. Find the development of one face of the cube, showing the penetration line.

3. A right cone, base 2″ diameter and altitude 3″, is penetrated by a cylinder of 1″.5 diameter. Their axes meet at right angles and both cylinder and cone rest on the H.P. Find the developments and penetration lines.

4. A cylinder of 1″.5 diameter penetrates another cylinder of 2″ diameter. Their axes meet at right angles. Find the penetration lines and their development.

5. Find the projections and show penetration lines and developments of two cylinders, as in Ex. 4, when the perpendicular distance between their axes is 0″.25, and the projections of their axes on a plane parallel to both make right angles.

6. A right cone of 3″ altitude, having a base of 2″ diameter, is penetrated by a sphere of 1″.5 diameter, so that the sphere touches the axis of the cone at a point 2″ from the apex. Show the penetration lines and their development.

CHAPTER VII.

AXOMETRIC PROJECTIONS.

WHEN it is desired to convey a clear idea of any structure or machine to those not skilled in the interpretation of drawings and at the same time to make a drawing which can be measured to scale, a method of representation on one plane, in which the dimensions are referred to three rectangular axes, is employed. If the axes are unequally inclined to the projection plane, the scales along them are unequal. The most usual arrangement, however, is to place the rectangular axes at the same inclination, in which case the scales are the same, and the projection is known as *isometric*. It must always be borne in mind that measurements can only be made in these projections along lines parallel to an axis.

The initial problem arising is either, given the projected directions of the axes to determine the scales, or, given the scales to determine the projection of the axes.

PROBLEM I.—Given the horizontal projections of the axes, —i.e., the projections of three straight lines which meet at a point and are at right angles to one another,—to determine their respective inclinations to the projection plane and hence the scales to be employed in the projection. (Fig. 79.)

Let the axes be AO, BO, and CO. Take any point *m* in one of the lines BO and draw *mn* perpendicular to AO.

mn will be parallel to the projection plane. Similarly draw *nr* and *rm* perpendicular to BO and CO respectively. Let a vertical plane contain AO and there will be formed a right-angled triangle *r*O*s* on the base *rs*. Rabat this triangle about *rs* parallel to the projection plane.

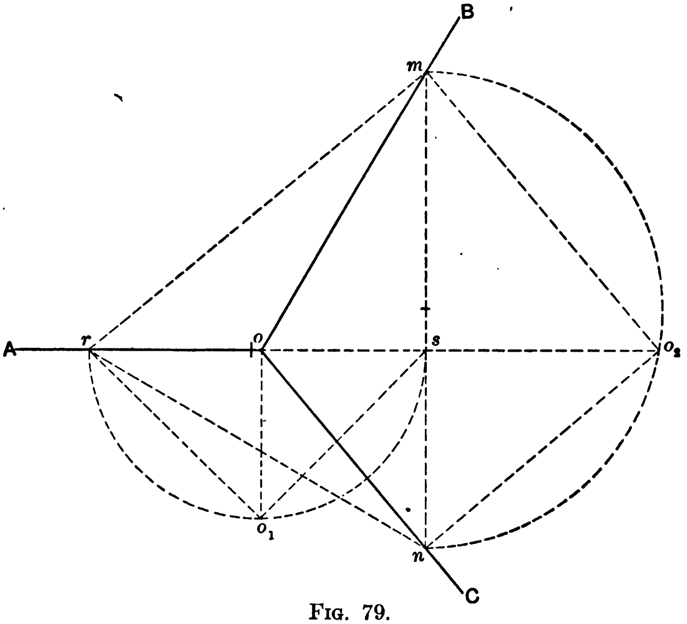

FIG. 79.

To do this, bisect *rs* and describe a semicircle on it; ro_1s will be the rabatted triangle, and oro_1 will represent the inclination of the axis AO. $\frac{ro}{ro_1}$ will be the scale of projection along this axis, and the scales along the other axes may be obtained similarly. To obtain the scales only and not the inclinations, it is slightly more convenient to rabat two of the right-angled triangles, as *rom* and *mon*, about *rm* and *mn*. Thus the scale along BO is $\frac{mo}{mo_2}$.

If the angles between the projection axes AO, BO, and CO are all equal (120°), then the axes must be equally inclined to the projection plane and hence the scales must

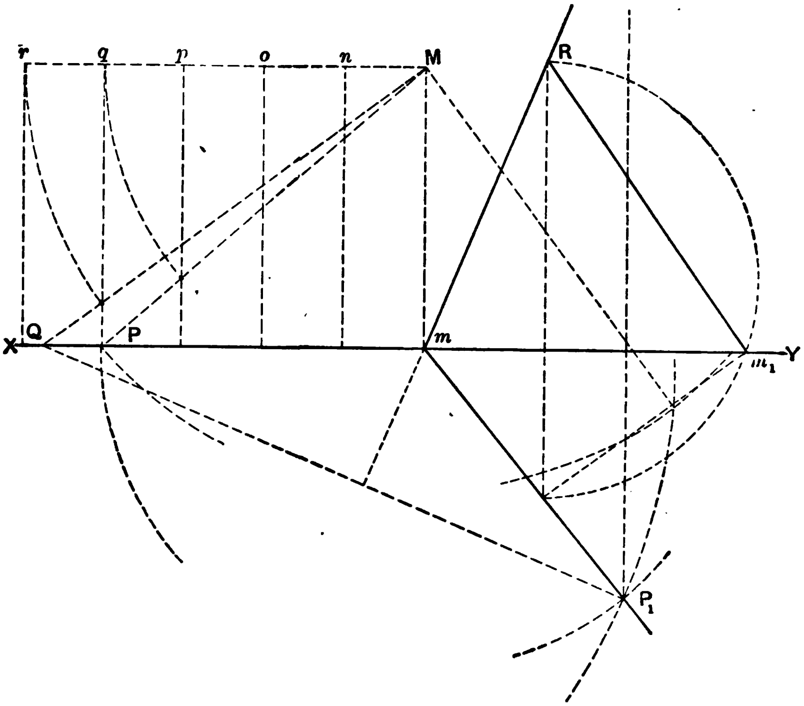

FIG. 80.

be equal, and the axes are those of an *isometric* (equal measure) projection.

PROBLEM II.—Given the scales along two of the axes and hence their inclinations to the projection plane, to determine the projections of the axes and the scale along the third axis. (Fig. 80.)

Let the given scales be $\frac{3}{4}$ and $\frac{4}{5}$. First determine the corresponding inclinations. Take a line parallel to XY at any convenient distance above XY. Divide the line into equal parts in n, o, p, etc., and by describing arcs about M with radii Mq and Mr obtain the inclinations MP and MQ corresponding to the scales $\frac{3}{4}$ and $\frac{4}{5}$ as in the figure. Now let M be a point in the V.P. and MP, MQ the inclinations of two lines including a right angle. Project the angle in QmP$_1$. (Prob. VIII, Chap. I.) QP$_1$ is a line in the horizontal projection plane, and the third axis is perpendicular to it. The scale along mR is obtained as in the previous problem and is $\frac{m\mathrm{R}}{m_1\mathrm{R}}$.

PROBLEM III.—Given the projections of the axes, to project a cube. (Fig. 81.)

The edges of the cube correspond to the axes of projection. Let oA, oB, and oC be the projections of the axes. Find the scale ratios as in Prob. I. From o_1 and o_2 measure the true lengths of the sides in o_2m_1, o_1n_1, o_1r_1, and transfer these lengths to the projected lengths along the axes in *om, on, or*. Hence construct the required projection of the cube, *omnrstuv*.

The scales for the construction along the axis may of course be reduced to any extent provided they are all reduced in the same ratio.

PROBLEM IV.—Given the projections of the axes, to project a cylinder of given dimensions. (Fig. 82.)

Draw a circle of the same diameter as the base of the cylinder, and enclose it in a square *abcd*. Let this square be the end of a prism having a length equal to the length of the cylinder. Project such a prism by the method of Prob. III. The circular end of the prism will touch the square which encloses it at the middle points o_1, q_1, etc.

It will be necessary to find the projections of certain additional points on the circle. These are to be found by references to the axes of projection. To find the intermediate points u, v, w, x, refer these to the containing sides of the square and measure a_1m_1, b_1n_1, b_1p_1, c_1r_1, etc., to the corresponding scales. Draw the corresponding lines through these points to determine the points $u_1v_1w_1x_1$ in the projection. Through these points and those previously determined sketch the ellipse. Draw a similar ellipse in the

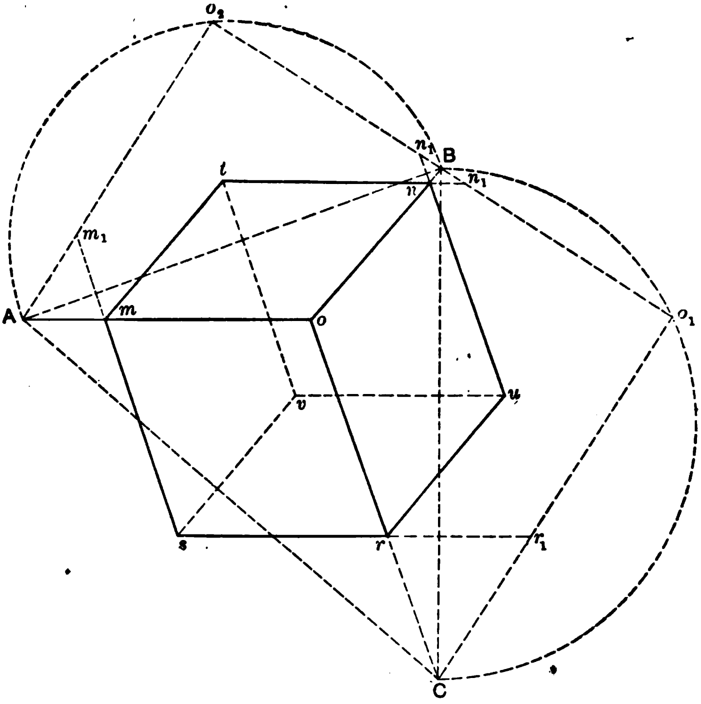

FIG. 81.

opposite end of the prism and draw the bounding lines to represent the outer generators of the cylinder.

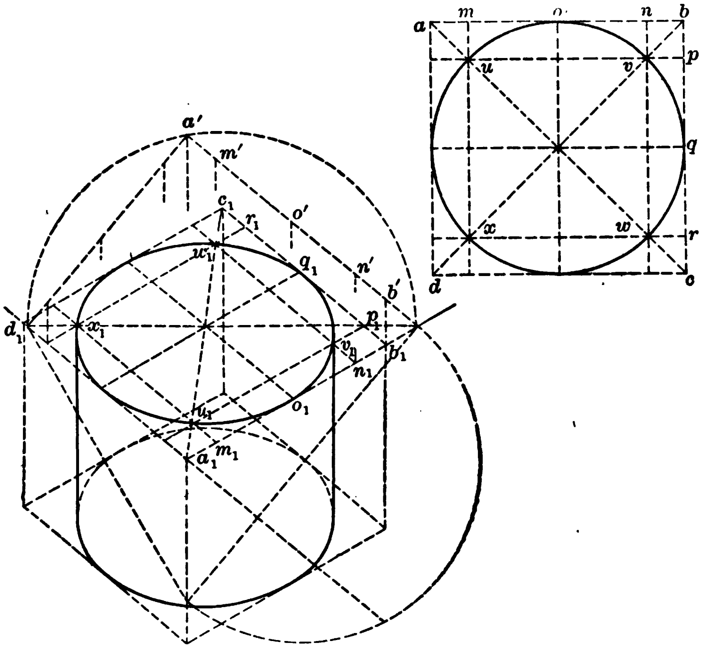

Fig. 82.

It is always to be borne in mind that measurements can only be made along lines parallel to an axis of projection and to the scale corresponding to that axis.

Exercises.

1. Find the isometric projection of a cube of 3″ edge.

2. Find the axometric projection of a cylinder of 2″ diameter and 3″ length when the scales along two of the axes are $\frac{3}{4}$ and $\frac{5}{6}$.

3. Make an orthographic projection of a mortise-and-tenon joint, and represent it in an isometric projection.

4. The axes of projection make angles of 120° and 135°. Represent a square pyramid, side of base 2″ and altitude 4″.

5. Make an isometric projection of a hexagonal nut for a bolt of 2″ diameter. The side of the hexagon may measure 1″.5 and the thickness of the nut also measure 1″.5.

CHAPTER VIII.

SHADES AND SHADOWS.

In order to give projections of objects the appearance of solidity they are sometimes represented as casting "shadows" on one or both of the projection planes, and those portions of them which are remote from the supposed source of light, i.e., the parts which the rays of light cannot reach, are represented as being in "shade."

The rays are usually supposed to come from an infinite distance, that is, they are parallel.

Problem I.—Given the direction of the rays, to find the shadow cast on the horizontal plane by a given line. (Fig. 83.)

Let AB be the line, and let the plan and elevation of a ray be represented by v and v'. Through A draw a line parallel to the given rays. The trace of this line in the H.P. will represent the shadow of A. Find this trace A_1 and similarly find the shadow of B in B_1. The line A_1B_1 will represent the required shadow of AB.

If the shadow on the V.P. were required, it would be found in a similar manner by determining the traces on that plane.

Problem II.—Find the shadow cast on the horizontal plane by a cube when two faces are horizontal and the other faces inclined to the V.P. (Fig. 84.)

Let v and v' represent the projection of the rays. By the method of Prob. I the shadow of the edge BC will be found in B_1C_1; the shadow of the vertical BG in B_1G_1; of GE in G_1E_1; of EF in E_1F_1; of the vertical DF in D_1F_1, and of DC in D_1C_1. Completing the shadow boundary line, the other corners will be found to have their shadows within this boundary. In the vertical projection the face DCHF might be represented as in shade.

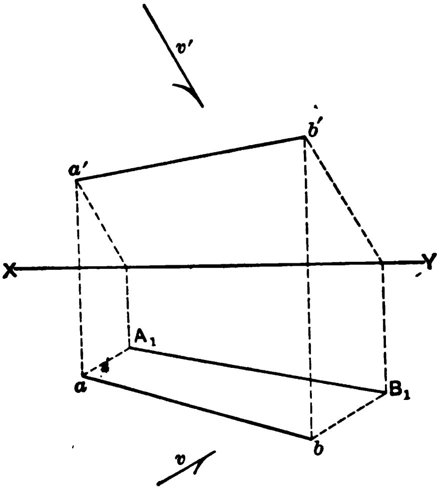

FIG. 83.

PROBLEM III.—Find the vertical projection of the shade and shadow on a cylindrical column surmounted by a rectangular cap. (Fig. 85.)

Let the direction of the rays be represented by v and v'. Let the projection of the lower edge of the cap be represented by *abcd*. First find the portion of the column which is in shade by drawing tangent lines to it parallel to the horizontal projection of rays. The front portion of the column in shade will lie to the right of x. Draw the vertical through x. To find the shadow cast on the cylindrical surface by *abcd,* imagine a vertical plane of rays passing through any point in the cap, as B. Such a plane will cut the cylinder

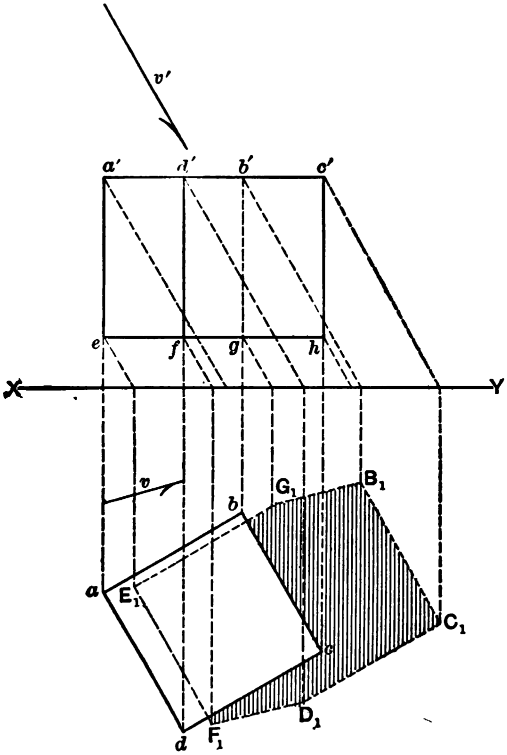

Fig. 84.

along a line of which the vertical projection is $u'u''$, and the ray passing through B and lying in this plane will meet the surface in a point whose vertical projection is u'', which therefore represents the shadow cast by B on the column. Similarly the shadows cast by E, F, G, and H are to be found in r'', s'', t'', u'', v'', w''. Join these freehand to represent the shadow boundary on the cylinder.

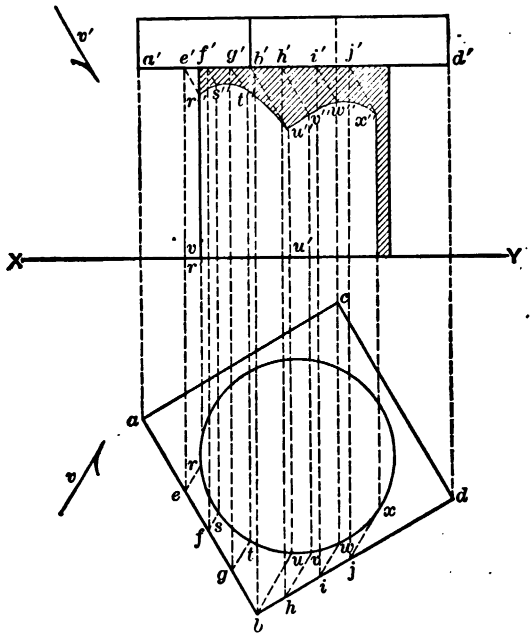

FIG. 85.

EXERCISES.

1. Find the shadow cast on the H.P. by a square pyramid standing on a square base. Find also the shadow on the surface of the base. Rays at 30° to the H.P. and 45° to the V.P. Let the altitude of the pyramid be 2″ and an edge of its base 1″; and let the base measure 3″×3″×1″ high.

2. A cross with its shaft vertical and its arms parallel to the vertical. Find the shadow on both H.P. and V.P. by rays of which the projections make angles of 45° with XY.

Let the height of the cross be 4″ and let the arms and upper portion measure 1″×½″×½″. The vertical shaft is also of material ½″×½″.

3. A cylindrical column 2″×1″ diameter with a plane cylindrical abacus 2″.5 diameter by ½″ deep. Find the shadow on both planes and the vertical projections of the shade and shadow on the object by rays of which the projections make angles of 60° with XY.

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